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Determination of the Modulus of Elasticity in Agricultural Seeds on the Basis of Elasticity Theory

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Abstract: Knowledge of apparent elastic properties such as Poisson's ratio and elastic modulus of agricultural seeds are important for the prediction of their load-deformation behavior and design of their process machines. The object of this study was investigated to test the applicability of three different methods existing in elasticity theory namely Hook's theory, Hertz theory and Boussinesq's theory for evaluation of the modulus of elasticity of agricultural seeds, as well as to study its general behavior under compressive loads and selection of the best theory among three reviewed theories. As a case study, the results of comparison elastic modulus of wheat determined on the basis of using above three theories were considered. The obtained results showed that Hertz theory (Parallel plates contact) is more suitable because of good agreement between theoretical and experimental results of force deformation curve. Also, between the four testing methods of loading of wheat the cylindrical indenter and core specimens give the lowest value while the parallel plate's method shows the highest values.

Key words: Elastic properties · Poisson's ratio · Elastic modulus · Hertz theory

INTRODUCTION

Recent activities in the mechanization of harvesting and handling of seeds and grains have brought a significant increase in research on mechanical properties of them. Totally, mechanical properties of agricultural seeds are needed for appropriate design of processing machines but its specific applications should be clearly understood before determining them experimentally. Literature review which has been done by the authors, of about twenty-five years, indicates extensive publication of many mechanical properties of various fruits, seeds and vegetables by agricultural engineers and food scientists. However, much of the past work dose not have a direct application toward understanding a phenomenon or designing a processing machine [1]. Thus it is prudent, first, to understand the need and then evaluate the required mechanical property.

The modulus of elasticity of an agricultural seeds is a mechanical property which has been suggested as a measure of the textural attribute designated as firmness. It is also an important property for determination of the stress cracks in agricultural seeds and usually calculates from force-deformation curve of agricultural seeds on the basis of elasticity theory. Thus, study of force-deformation behavior of agricultural seeds in their natural state has been an attempt to provide objective measurements resulting in more meaningful data usable in engineering analysis and design. Despite the complexity of the material and the test specimens and the lack of knowledge of stress-strain distribution within the usually convex body, this approach has been justified on the ground that the raw material is usually subjected to mechanical treatments in handling and processing in its natural form.

Meantime, one of the best problems in using elasticity theory for determining of the modulus of the elasticity of agricultural seeds is that agricultural seeds display characteristics of both elastic solids and viscous liquid and is called viscoelastic. However, many researches have found that when the small loads are occurred in short times, application of elasticity theory for agricultural seeds are applicable [2-7].

Many researches have determined the modulus of the elasticity by using of elasticity theory [5-16]. Many of these studies have made use of the Boussinesq and Hertz

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theories in an attempt to provide better defined parameters to express resistance to mechanical damage, describe some of the mechanical attributes of food texture, such as firmness and hardness.

The objective of the present work was to test the applicability of three different theories for evaluation of the modulus of elasticity of agricultural seeds, as well as to study its general behavior under compressive loads.

MATERIALS AND METHODS

Uniaxial Compression: As it was discussed in the last part, one of the best techniques to determine the modulus of elasticity in agricultural seeds is studying their behavior using force-deformation curve of these materials. Meantime, the various tests are available for taking force-deformation curve of agricultural seeds such as compression, tensile and bending tests (Table 1). Of course, because of several limitations, very few investigators have tried the tensile and bending tests for agricultural seeds. Problems of the preparation of test specimen and complications induced by the gripping of the specimen for tensile test on the one hand and the fact that mechanical damage to the produce usually results from compressive loads have probably been the reasons for finding more data reported for compressive tests than other tests [1]. Generally, compassion tests data are used with three elastic theories of contacting bodies: Hook, Hertz and Boussinesq theories.

Hook's Theory: For ideal elastic materials, stress (σ) is directly proportional to strain (ε) and Young's modulus (E) based on Hook's law [17] is given by Eq. (1). Young's modulus is properly determined from the initial section of the stress-strain curve at relatively low deformation. Thus only the section of the deformation curve below the initial yield point is considered and the value obtained is referred to as the elastic modulus:

$$E = \frac{\sigma}{\varepsilon} = \frac{Pl}{\delta A} \quad (1)$$

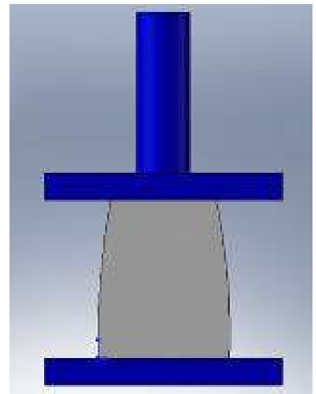

Fig. 1. Cylindrical specimen – Hook's theory

In order to use this theory, agricultural seeds should be shaped at both ends so that cylindrical specimens obtained. Then, the cylindrical specimens are supposed to certain load for attaining force-deformation curve (Fig. 1). Lastly, with knowing the values of load used (P), elastic deformation (δ), initial height of the specimen(l) and contact area of specimen (A), modulus of the elasticity are calculated from Eq. (1) [11]. Shelef and Mohsenin [5] used this theory for determination of elastic modulus of wheat. They reported that for obtaining good results in application of this theory for agricultural seeds, the proportion of height of specimen to its wide should be a number between 2 to 10. They also revealed that this theory is not appropriate for determination of elastic modulus of agricultural seeds.

Hertz Theory: In 1896 Heinrich Hertz [18] proposed a solution for contact stresses in two elastic isotropic bodies, such as the case of two spheres of the same material touching each other. In this problem, Hertz attempted to find answers to such questions as the form of the surface of pressure, the magnitude of the curve of pressure, normal pressure distribution on the surface of pressure, the magnitude of the maximum pressure and the approach of the centers of the bodies under pressure. Hertz's theory for contact stresses between two elastic bodies subjected to uniaxial compression, as reviewed by

Table 1: Usual tests used for modulus of elasticity determination

Test used	Material used	Reference
Compression test	Apple, corn, peach, tomato, soybean and wheat	Arnold and Mohsenin (1971), Shelef and Mohsenin (1969), Fridley *et al*. (1968), Jindal and Techasena (1985), Misra and Young (1981), Arnold and Roberts (1969)
Tensile test	Cherry and tomato	Levin *et al*. (1959), Huff (1967)
Bending test	Corn and pea	Balastreire *et al*. (1982), Khzaie (2002)

Table 2: Force-deformation methods used for modulus of elasticity determination

Theory used	Material used	Type of specimen	Type of loading	Reference
Hook's Law	Apples, potatoes, wheat, corn	Cylindrical, core.	Flat plate	Arnold and Roberts (1969), Finney *et al*. (1967), Mohsenin and Cooper (1963), Mohsenin and Cooper (1965), Shelef and Mohsenin (1969), Zoreb and Hall (1960)
Hertz	Apples, peaches, pears, corn, wheat	Whole fruit, fruit segments; skin on, skin off. Whole grains, core slabs.	Flat plat, single and double contact, Spherical indenter	Arnold and Roberts (1966), Arnold and Roberts (1969), Finney *et al*. (1967), Mohsenin and Cooper (1963)
Boussinesq	Apples, peaches, pears, potatoes, wheat, corn	Whole fruit, fruit segments; skin on, skin off. Whole grains, core slabs.	Cylindrical die	Finney *et al*. (1967), Morrow and Mohsenin (1966), Shelef and Mohsenin (1969), Timbers *et al*.(1965)

Kosma and Cunningham [19], Kiani *et al*. [6] and Mohsenin [1] was employed for calculation of the modulus. According to this theory, the deformation of the two convex bodies is given by

$$D = \frac{K}{2}\left[\frac{9}{16\pi^2}P^2(Q_1+Q_2)^2\left(\frac{1}{R_1}+\frac{1}{R_1'}+\frac{1}{R_2}+\frac{1}{R_2'}\right)\right]^{1/3} \quad (2)$$

Where:
D = Approach of body centers;
K = Constant determined from elliptic integral tables;
P = Applied compressive load;

$$Q = \frac{4(1-\mu^2)}{E}$$

μ = Poisson' s ratio;
E = Elastic modulus,
R = Major radius of curvature;
R' = minor radius of curvature;
- Denotes primary convex body
- Denotes secondary convex body

The use of this equation requires that eight fundamental assumptions be satisfied, which have been listed in detail by Kozma and Cunningham [19]. The first known application of the Hertz theory for contact stresses in agricultural seeds is reported by Shpolyanskaya [20] for determination of modulus of deformability of the wheat grain compressed between two parallel plates. Application of this assumption to agricultural seeds was discussed by Morrow and Mohsenin [3]. Meantime, many researches have revealed that when piles of agricultural seeds in the form of spherical bodies are considered, the Hertz method can be used to determine the contact forces, displacements on individual units and finally elastic modulus [5, 11, 12, 21]. They used this theory for agricultural seeds in two states depend on specimen's condition, type and shape of loading. They also simplified Eq. (2) to yield an expression for calculation of the modulus in each state.

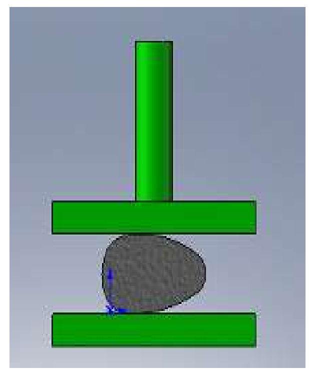

Fig. 2: Parallel plates contact (whole specimen) – Hertz theory

Parallel plates (whole specimen)

In this state, whole specimen is fixed to a metal plate and then a compressive load is applied by means of parallel plates (Fig. 2). For first time, Morrow and Mohsenin [3]; Arnold and Roberts [4] discussed the use of the Hertz theory for plate type tests on fruits and wheat germs, respectively. Since the flat plate results in a flat plane across the area of contact with specimen, Eq. (2) can be simplified to describe the plate test in either one of two ways. First, the equation may be reduced by assuming that two identical spheres pressed together are mirror images and deform equal amounts, second, the plate can be assumed to be a sphere of infinite radiuses with a modulus of elasticity which is very great compared to that of a specimen and all deformation can be assumed to occur within the specimen. Both ways of reasoning can be used to reduce the equation to:

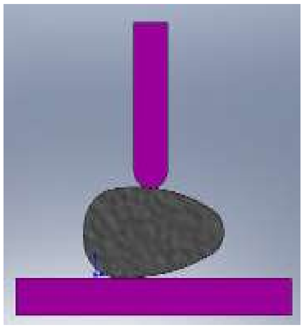

Fig. 3: Spherical Indenter Contact (Whole specimen) – Hertz theory

$$E = \frac{0.5P\left(1-\mu^2\right)}{\Delta L^{\frac{3}{2}}}\left(\frac{1}{R}+\frac{1}{R'}\right)^{\frac{1}{2}} \qquad (3)$$

It should be mentioned that many researchers have used this method to determine the elastic modulus of agricultural seeds [11, 21, 22, 23].

Spherical Indenter Contact: Timbers *et al*. [24] have reported that a measurement of elastic modulus can be obtained by application of the theory o f elasticity to an Indenter contact test on the cheek of fruits. The results reported derived from the case of a rigid die in the form of a circular cylinder (Indenter) pressed against the plain boundary of a semi-infinitive elastic solid. Shelef and Mohsenin [21] used this method for determination of elastic modulus of wheat. In their research a compressive load was applied by means of a steel spherical indenter having a 0.16 mm diameter. The elastic modulus of wheat was obtained 6015 MPa in their research. They have reported that for obtaining good results in using of this method, the proportion of diameter of indenter to diameter of specimen should be a number between 0.066 and 0.071. Arnold and Mohsenin [12] used a steel spherical indenter having a 1.59 mm diameter for wheat. This method for agricultural seeds is used in two states depending on specimen's condition. They also simplified Eq. (2) to yield an expression to calculate the modulus in each state as below:

Whole Specimen (Fig. 3):

$$E = \frac{0.5P\left(1-\mu^2\right)}{\Delta L^{\frac{3}{2}}}\left(\frac{1}{R}+\frac{1}{R'}+\frac{4}{d}\right)^{\frac{1}{2}} \qquad (4)$$

Where d is the diameter of the spherical indenter.

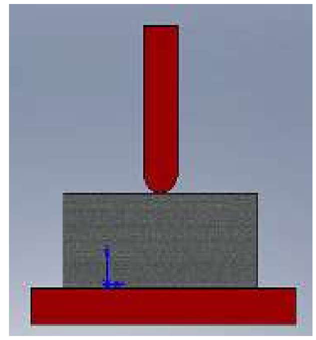

Fig. 4: Spherical Indenter Contact (Flat specimen) – Hertz theory

Flat Specimen (Fig. 4)

$$E = \frac{0.5P\left(1-\mu^2\right)}{\Delta L^{\frac{3}{2}}}\left(\frac{4}{d}\right)^{\frac{1}{2}} \qquad (5)$$

Boussinesq's Theory: For evaluating mechanical properties of food materials such as fruits and vegetables and cereal grains several investigators have used a rigid cylindrical die to study load-deformation behavior of the material. This method of testing, as suggested by Finney and Hall [25], approximated the Boussinesq's problem in which a rigid die, in the form of a circular cylinder, is pressed against the plane boundary of a semi-infinite elastic solid. The original solution of this problem by Boussinesq [26] has been expanded by Timoshenko and Goodier [27] to include three dimensional considerations, while the Hertz method of analysis may be suggested for determination the maximum surface pressure and elastic modulus when one convex body is pressed against another convex body or against a flat plate, the Boussinesq method may be applied to study the pressure distribution under a rigid die as well as the evaluation of elastic modulus. Timoshenko and Goodier [27] proved that with knowing the values of load used (P), elastic deformation (δ), Poisson's ratio (μ) and the diameter of the die (2a), modulus of the elasticity can be calculated from Eq. (6):

$$E = \frac{P\left(1-\mu^2\right)}{2a\delta} \qquad (6)$$

Arnold and Roberts [11] used this theory for determination of elastic modulus of wheat. They reported that for obtaining good results in application of this theory for agricultural seeds, the diameter of the die should be smaller than the diameter of the specimen.

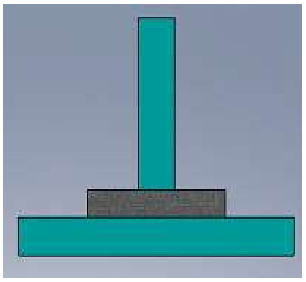

Fig. 5: Flat Specimen – Boussinesq theory

So, they revealed that this theory is not very agreeable for determination of elastic modulus of agricultural seeds. In order to use this theory, agricultural seeds should be shaped at both ends so that flat specimens obtained (Fig 5).

RESULTS AND DISCUSSION

Comparison of Modulus of Elasticity of Wheat Determined on the Basis of Using above Three Theories: Table 3 shows the value of modulus of elasticity of wheat reported by several sources. The methods of tests are indicated as: (1) loading the grain in flat position between parallel plates, (2) loading in flat position by a spherical indenter (3) loading in flat position by a rigid cylindrical die and (4) by cutting the ends of the grain off and loading the core specimen as a "cylinder". When the four testing methods of loading are compared, the cylindrical indenter and core specimens give the lowest value while the parallel plate's method shows the highest values. The low value for the core specimens is predictable, since in this case the structural mechanics of the grain was changed by removing both ends of the specimen. When the grain is compressed by the cylindrical indenter, a full contact area is immediately reached and the displacement is constant over the circular base of the cylinder. On the other hand, under the both parallel plates method the area of pressure is gradually increasing with loading and the case is more complex. To confirm this justification, Fridley *et al*. [10] concluded that plate test is preferable to indenter test for agricultural seeds because the test procedure is less critical and results compare more favorable with those predicted from theory. Furthermore, this type of loading is more representative of the type of load application found in practice.

Selection of the Best Theory among Three Reviewed Theories for Determination of Modulus of Elasticity of Agricultural Seeds: In general, the authors could not find any published work to recommend one of the existing theories as the best one for agricultural seeds. Arnold and Roberts [11] who applied Hertz theory to compress cubical samples of wheat revealed that to distinguish one theory as the more precise method to determine modulus elasticity of grain such as wheat is impossible. Shelf and Mohsenin [21] also did not suggest any particular method for this purpose. However, the study on obtained force-deformation curves of a variety of seeds using different theories led the authors to the following consequences for selection the appropriate method to determine modulus of elasticity of seeds:

- Hertz theory is more suitable because of good agreement between theoretical and experimental results of force deformation curve [1, 11, 12].
- Applying Boussinesq theory in compression test using indenter will lead to almost a nonlinear behavior of force-deformation curve in its elastic part
- Arnold and Roberts [11] suggested the method of using logarithmic scale and the comparison of obtained line's inclination to a line with inclination of 1.5.

Table 3: Modulus of elasticity for wheat grain under uniaxial compression

Wheat variety	Loading rate, in/min	Moisture content (%)	Testing method	Modulus of elasticity 10^5 psi	Source
Seneca	0.02	10 (d.b.)	Parallel plate- whole grains	4.12	Shelef abd Mohsenin (1969)
			Spherical indenter- whole grains	8.30	
			Cylindrical indenter - whole grains	1.57	
			Parallel plate-core specimens	2.30	
Lyntestens 62		11 – 12	Parllel plate-whole grains	21.30	Shpolyanskaya (1952)
Gordeiforme 10	–	11 –12	Parllel plate-whole grains	28.40	Zoreb (1960)
Soft red winter	0.08 – 0.5	15.7 (d.b.)	Parallel plates-core specimens	0.46	
Seven Australian variety	0.26	11.5 – 13 (w.b.)	Parllel plate-whole grains	17.4-5.95	Arnold and Roberts (1967)
	`		Parallel plate-core specimens	20.5-4.11	

- Although using intender with spherical end and specified diameter is suggested for Hertz theory, parallel plates should be used for compression test of seeds because of some limitations such as the possibility of seed's grinding, the effect of intender's diameter and the sensitivity of contact point on the final value of modulus of elasticity. This result was also revealed by Arnold and Roberts [11].
- Using Hertz and Boussinesq's theories needs more assumptions compared with the other theories.

Poisson's Ratio of Agricultural Seeds: As it was talked in last parts, using of Hertz and Boussinesq theories for determination of elastic modulus is required to knowing Poisson's ratio. So, many researchers have studied Poisson's ratio for agricultural produce. In a study on wheat that was done by Shelef and Mohsenin [5], Poisson's ratio was assumed 0.4. Arnold and Roberts [4] have investigated the effect of deformation and elastic modulus on Poisson's ratio of wheat. They revealed that variation of Poisson's ratio of wheat was 0.3 to 0.5. Many researchers have also assumed a value of 0.4 for Poisson's ratio of agricultural produce [5, 10, 13].

Recommendations: The risk of damage and cracking of seeds during harvesting, conveying and processing is high. Therefore, to analyze the effect of cracking on loosing the resistance of seeds to fracture, it is necessary to determine the fracture characteristics. There is a simple relationship between fracture characteristics and modulus of elasticity (E), so it can be estimated if the fracture parameters are available. Although the modulus elasticity can be determined using different techniques, determination of fracture characteristics is the main benefit of applying fracture method.

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