

## De Sitter Super Algebra

*Ali Pahlavan and Amad ZatiRostami*

Department of Physics, Islamic Azad University, Sari branch,  
 P.O. Box: 48175-914, Sari, Mazandaran, Iran

---

**Abstract:** We discuss the de Sitter group  $SO(1,4)$ , i.e. space-time symmetry of de Sitter space and its universal covering group  $Sp(2,2)$ . We recall the transformation properties of the spinor fields  $\psi^{(x)}, \bar{\psi}^{(x)}$ . The charge conjugation symmetry of the de Sitter spinor field is presented in ambient space notation. The de Sitter super algebra discussed in coordinate independent way.

**Key words:** Missing

---

### INTRODUCTION

Recent astrophysical data coming from type Ia supernovas indicate that our universe might currently be in a de Sitter (dS) phase. Therefore, it is important to find a formulation of de Sitter quantum field theory with the same level of completeness and rigor as for its Minkowskian counterpart. But a number of arguments are usually put forward for the non-existence of supersymmetry models with a positive cosmological constant, i.e. supersymmetry in de Sitter space. Such arguments are often based on the non-existence of Majorana spinors for  $O(4,1)$ . Indeed, one can use the "Noether coupling" approach to super gravity to directly show that Majorana gravitini are incompatible with a positive cosmological constant. However, there is no need to insist on the existence of Majorana spinors. We may simply accept that for every spinor its charge-conjugate is exist and it is independent [1-4].

In the previous paper, the charge-conjugate spinor obtained. In this paper, the supersymmetry in de Sitter space is studied in the ambient space notation. In the next section firstly, we discuss the de Sitter group  $SO(1,4)$ , i.e. space-time symmetry of de Sitter space and its universal covering group  $Sp(2,2)$ . We recall the transformation properties of the spinor fields  $\psi^{(x)}, \bar{\psi}^{(x)}$ . The charge conjugation symmetry of the de Sitter spinor field in ambient space notation presented in section 3. The most general de Sitter superalgebra is presented in section 4. Section 5 is devoted to the the structure of the internal symmetry. Finally, a brief conclusion and outlook are given in Section 7.

**De Sitter Group:** The de Sitter space is an elementary solution of the positive cosmological Einstein equation in the vacuum. It is conveniently seen as a hyperboloid embedded in a five-dimensional Minkowski space.

$$X_H = \{x \in \mathbb{R}^5; x^2 = \eta_{\alpha\beta} x^\alpha x^\beta = -H^{-2}\}, \alpha, \beta = 0, 1, 2, 3, 4, \quad (1)$$

Where  $\eta_{\alpha\beta} = \text{diag}(1, -1, -1, -1, -1)$ . The de Sitter metrics reads

$$ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta = g_{\mu\nu}^{dS} dX^\mu dX^\nu, \mu = 0, 1, 2, 3, \quad (2)$$

Where the  $X^\mu$  's are the 4 space-time intrinsic coordinates in dS hyperboloid. Different coordinate systems can be chosen. The kinematical group of the de Sitter space is the 10-parameter group  $SO_0(1,4)$  and its contraction limit  $H=0$  is the Poincar'e group. There are two Casimir operators

$$Q^{(1)} = -\frac{1}{2} L_{\alpha\beta} L^{\alpha\beta} \quad (2)$$

$$Q^{(2)} = -W_\alpha W^\alpha, \quad W_\alpha = -\frac{1}{8} \epsilon_{\alpha\beta\gamma\delta\eta} L^{\beta\gamma} L^{\delta\eta}, \quad (2)$$

Where  $\epsilon_{\alpha\beta\gamma\delta\eta}$  is the usual antisymmetrical tensor and the  $L_{\alpha\beta}$ 's are the  $(\psi_c)_c = C\gamma^\beta \gamma^\alpha \psi_c^* = C\gamma^\beta \gamma^\alpha (C^* \gamma^\beta \gamma^\alpha \psi) = \psi$ . infinitesimal generators, which obey the commutation relations

$$[L_{\alpha\beta}, L_{\gamma\delta}] = -i(\eta_{\alpha\gamma} L_{\beta\delta} + \eta_{\beta\delta} L_{\alpha\gamma} - \eta_{\alpha\delta} L_{\beta\gamma} - \eta_{\beta\gamma} L_{\alpha\delta}). \quad (3)$$

The infinitesimal generators  $L_{\alpha\beta}$  can be represented as  $L_{\alpha\beta} = M_{\alpha\beta} + S_{\alpha\beta}$ , where

$$M_{\alpha\beta} = -i(x_\alpha \partial_\beta - x_\beta \partial_\alpha) \quad (4)$$

and

$$S_{\alpha\beta} = -\frac{i}{4}[\gamma_\alpha, \gamma_\beta] \quad (5)$$

are respectively the "orbital" and "spinorial" parts of  $L_{\alpha\beta}$ . The five  $4 \times 4$  matrices  $\gamma^\alpha$  are the generators of the Clifford algebra based on the metric  $\eta_{\alpha\beta}$ :

$$\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2\eta^{\alpha\beta} 1, \quad \gamma^{\alpha\dagger} = \gamma^0 \gamma^\alpha \gamma^0. \quad (6)$$

An explicit (and convenient for the sequel) representation is provided by [1-4]

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \gamma^2 = \begin{pmatrix} 0 & i\sigma^1 \\ i\sigma^1 & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}, \gamma^4 = \begin{pmatrix} 0 & i\sigma^3 \\ i\sigma^3 & 0 \end{pmatrix} \quad (7)$$

in terms of the  $2 \times 2$  unit 1 and Pauli matrices  $\sigma^i$ . This representation will prove to be useful when discussing the physical relevance of group element decomposition.

The spinor wave equation in de Sitter space-time has been originally deduced by Dirac in 1935 [7] and can be obtained starting from the eigenvalue equation for the second order Casimir operator [6]

$$(-ix\gamma \cdot \bar{\partial} + 2i + v)\psi(x) = 0, \quad (8)$$

Where  $x = \eta_{\alpha\beta} \gamma^\alpha x^\beta$  and  $\bar{\partial}_\alpha = \partial_\alpha + x_\alpha x \cdot \partial$ . Because of the de Sitter group covariant, the adjoint spinor is defined as follows:

$$\bar{\psi}(x) \equiv \psi^\dagger(x) \gamma^0 \gamma^4. \quad (9)$$

Let us now recall also the transformation properties of the spinor fields  $\psi(x), \bar{\psi}(x)$ . The two-fold and universal covering group of  $SO_0(1,4)$  is the (pseudo-)symplectic group  $Sp(2,2)$ ,

$$Sp(2,2) = \left\{ g \in Mat(2;H) : \det g = 1, g^\dagger \gamma^0 g = \gamma^0 \right\}, \quad (10)$$

Where  $g^\dagger = {}^T \bar{g}$ ,  ${}^T g$  is the transposed of  $g$  and  $\bar{g}$  the quaternionic conjugate of  $g$ . For obtaining the isomorphism relation between them defines the matrices  $X$  associated with  $x \in X_H$  by:

$$X = \begin{pmatrix} x^0 & P \\ \tilde{P} & x^0 \end{pmatrix}, \quad (11)$$

Where

$$P = (x^4, \bar{x}) = x^4 1 + ix^1 \sigma^1 - ix^2 \sigma^2 + ix^3 \sigma^3, \quad (12)$$

is a quaternion and  $\tilde{P} = (x^4, -\bar{x})$  is their quaternion conjugate. By the representation (7) of the  $\gamma$  matrices, one can write  $X$  in the following form:

$$x = x \cdot \gamma = X \gamma_0 = \begin{pmatrix} x^0 & -P \\ \tilde{P} & -x^0 \end{pmatrix}. \quad (13)$$

The transformation of  $X$  under the action of the group  $Sp(2,2)$  is

$$X' = g X g^{-1}, \quad x' = g x g^{-1}. \quad (14)$$

For  $\Lambda \in SO_0$  and  $g \in Sp(2,2)$  we have

$$\begin{aligned} x'^\alpha &= \eta^{\alpha\beta} x'_\beta = \frac{1}{4} \text{tr}(\gamma^\alpha \gamma^\beta) x'_\beta = \frac{1}{4} \text{tr}(\gamma^\alpha g x g^{-1}) \\ &= \frac{1}{4} \text{tr}(\gamma^\alpha g \gamma^\beta g^{-1}) x_\beta = \Lambda^{\alpha\beta}(g) x_\beta. \end{aligned} \quad (15)$$

Therefore, for all  $g \in$ , it correspond a transformation  $\Lambda \in S_{0,}(1,4)$ ,

$$\Lambda^\alpha_\beta(g) = \frac{1}{4} \text{tr}(\gamma^\alpha g \gamma_\beta g^{-1}), \quad \Lambda^\alpha_\beta \gamma^\beta = g \gamma_\alpha g^{-1}, \quad (16)$$

which is the isomorphism relation between two group

$$SO_0(1,4) \sim Sp(2,2)/2 \quad (17)$$

The transformation laws for the  $\psi(x)$  and its adjoint  $\bar{\psi}(x)$ , under which the de Sitter-Dirac equation is covariant, are:

$$\psi(x) \rightarrow \psi'(x) = g^{-1} \psi(\Lambda(g)x), \quad (18)$$

$$\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(\Lambda(g)x) i(g), \quad (19)$$

Where  $i(g) = -\gamma^4 g \gamma^4$  is a group involution in  $Sp(2,2)$ . Similar to the Minkowskian space, we can define  $g$  by

$$g = \exp\left[-\frac{i}{2} \omega^{\alpha\beta} S_{\alpha\beta}\right], \quad \omega^{\alpha\beta} = -\omega^{\beta\alpha}. \quad (20)$$

It is satisfy  $\gamma^0 g^\dagger \gamma^0 = g^{-1}$ , i.e.  $g \in Sp(2,2)$ .

**Charge conjugation**

In order to obtain the Minkowskian charge conjugation in the null curvature limit, the charge conjugation spinor  $\psi^c$  is defined as [1-4]

$$\psi^c = \eta_c C(\gamma^4)^T (\bar{\psi})^T, \tag{21}$$

Where  $\eta_c$  is an arbitrary unobservable phase value, generally taken as being equal to unity. In the present framework charge conjugation is an antilinear transformation. In the  $\gamma$  representation (7) we have:

$$\begin{aligned} C\gamma^0 C^{-1} &= -\gamma^0, C\gamma^4 C^{-1} = -\gamma^4 \\ C\gamma^1 C^{-1} &= -\gamma^1, C\gamma^3 C^{-1} = -\gamma^3, C\gamma^2 C^{-1} = \gamma^2. \end{aligned} \tag{22}$$

In this representation C commute with  $\gamma^2$  and anticommute with other  $\gamma$ -matrix therefore the simple choice may be taken as  $C=\gamma^2$ . It satisfies

$$C = -C^{-1} = -C^T = -C^\dagger. \tag{23}$$

This clearly illustrates the non-singularity of C.

The adjoint spinor, which is defined by  $\bar{\psi}(x) \equiv \psi^\dagger(x)\gamma^0\gamma^4$ , transforms in a different way from  $\psi$ , under de Sitter transformation. On the contrary, it is easy to show that  $\psi^c$  transforms in the same way as  $\psi$ ,

$$\psi'^c(x') = S(\Lambda)\psi^c(x).$$

The wave equation  $\psi^c$  of is different from the the wave equation of  $\psi$  by the sign of the q and v. Thus it follows that if  $\psi$  describes the motion of a dS-Dirac particle with the charge q,  $\psi^c$  represents the motion of a dS-Dirac anti-particle with the charge (-q). In other words  $\psi$  and  $\psi_c$  can describe "particle" and "antiparticle" functions.  $\psi$  and  $\psi_c$  are charge conjugation of each other

$$(\psi_c)_c = C\gamma^0\gamma^4\psi_c^* = C\gamma^0\gamma^4(C^*\gamma^0\gamma^4\psi) = \psi. \tag{24}$$

**Super Algebra:** Supersymmetry in spaces of constant curvature has been considered recently. In this section, we follow the supersymmetry algebra, which was constructed by [1-4]. For extension of the de Sitter group, the generators of supersymmetry transformation  $Q_i^r, i=1,2,3,4; r=1, \dots, N$  are introduced.  $Q_i^r$  are fermionic

generators which transform as spinors under the de-Sitter group. The super-algebra, which construct by the generators  $L_{\alpha\beta}$  and  $Q_i^r$ , is not closed. If we present an internal symmetry with the generator  $T_a$  that commute with de Sitter generator it is possible to find a closed super-algebra. Therefore the de Sitter super-algebra in four-dimensional space-time has the following generators:

- The generators  $L_{\alpha\beta}$  of the Lie algebra  $So(1,4)$ , which obey the commutation relations (3).
- The internal group generators are defined by

$$T_a \quad a=1,2,\dots,n$$

- The 4 -component dS-Dirac spinorial generator are defined by

$$Q_i^r, \quad i = 1,2,3,4, r = 1,2,\dots, N$$

Where i is the spinorial component and r is the supersymmetry index.

It is possible to find a closed algebra with the following relation:

$$[L_{\alpha\beta}, L_{\gamma\delta}] = -i(\eta_{\alpha\gamma}L_{\beta\delta} + \eta_{\beta\delta}L_{\alpha\gamma} - \eta_{\alpha\delta}L_{\beta\gamma} - \eta_{\beta\gamma}L_{\alpha\delta}), \tag{25}$$

$$[T_a, T_b] = C_{ab}^d T_d, \tag{26}$$

$$\{Q_i^r, Q_j^s\} = G_{ij}^{rs} Q_i^r + D_{ij}^{rs\alpha\beta} L_{\alpha\beta} + E_{ij}^{rs} T_a, \tag{27}$$

$$[L_{\alpha\beta}, T_a] = 0 \tag{28}$$

$$[Q_i^r, L_{\alpha\beta}] = A_{ir}^{\alpha\beta} Q_i^r + B_{i\alpha\beta}^{\alpha'\beta'} L_{\alpha'\beta'}, \tag{29}$$

$$[Q_i^r, T_a] = F_{iar}^{\alpha\beta} Q_i^r, \tag{30}$$

Where A,B,C,D,E,F and G are constant. The values of these constants determine completely the de Sitter supersymmetry algebra. The constant C defined the structure of the internal symmetry. In the following by the use of the different Jacobi identities, these constants can be defined [1-4].

Since, the direct product of the two spinors is transform as a tensor, therefore G=0. We can say that in (29) whenever the direct product of the tensor with a spinor, the results is the spinor not tensor, therefore B=0. Consequently, we have:

$$\{Q_i^r, Q_j^l\} = D_{ij}^{rl\alpha\beta} L_{\alpha\beta} + E_{ij}^{rla} T_a, \quad (31)$$

$$[Q_i^r, L_{\alpha\beta}] = A_{ir'}^{\alpha\beta} Q_i^{r'}, \quad (32)$$

Due to the equations (25) and (28), A can be written in terms of new matrix  $\Gamma$ :

$$A_{ir'}^{\alpha\beta} = \frac{1}{2} (\Gamma_{\alpha\beta})_i^j \delta_{r'}^r, \quad (33)$$

Where  $\Gamma_{\alpha\beta} = \Gamma_{\alpha\beta}$  By the anti-commute (27) and relation  $\{Q_i^r, Q_j^l\} = \{Q_j^l, Q_i^r\}$  we obtain

$$D_{ij}^{rl\alpha\beta} L_{\alpha\beta} + E_{ij}^{rla} T_a = D_{ji}^{lr\alpha\beta} L_{\alpha\beta} + E_{ji}^{lra} T_a,$$

consequently  $D_{ij}^{rl\alpha\beta} = D_{ji}^{lr\alpha\beta}$  and  $E_{ij}^{rla} = E_{ji}^{lra}$ . The Jacobi identity (L,Q,Q) is provided us to get

$$D_{ij}^{rl\alpha\beta} = \omega^{rl} (\Gamma^{\alpha\beta} C^{-1})_{ij}, \quad E_{ij}^{rla} = w^{ar} (C^{-1})_{ij},$$

Where  $C = \gamma^2$  is the charge conjugation and  $\omega_i = \omega_r$  and  $w_{rl}^a = -w_{lr}^a$ . We observe that in the  $\{Q,Q\}$  anticommutator the internal symmetry generators appears in the form.

$$T_{rl} = w_{rl}^a T_a, \quad T_{rl} = -T_{lr}$$

and we will use  $T_{rl}$  rather than  $T_a$  to determine the structure of the algebra. For this we multiply the generators  $T_a$  in (26), (28) and (30) with  $w_{ij}^a$ . The (Q,Q,Q) Jacobi identity,

$$\{\{Q_i^r, Q_j^l\}, Q_k^p\} + \{\{Q_k^p, Q_i^r\}, Q_j^l\} + \{\{Q_j^l, Q_k^p\}, Q_i^r\} = 0,$$

uniquely determines the commutator [Q,T],

$$[Q_i^r, T^{lp}] = -2(\omega^{rl} Q_i^p - \omega^{lp} Q_i^r). \quad (34)$$

The (Q,Q,T) Jacobi identity,

$$[\{Q_i^r, Q_j^l\}, T_{pm}] + [[T_{pm}, Q_i^r], Q_j^l] + \{[Q_j^l, T_{pm}], Q_i^r\} = 0.$$

gives the following commutator

$$[T_{rl}, T_{pm}] = -2(\omega_{lp} T_{mr} + \omega_{rm} T_{lp} - \omega_{rp} T_{lm} - \omega_{lm} T_{pr}),$$

Where we have used the antisymmetry condition of T. Now we can write the full superalgebra in the form

$$[L_{\alpha\beta}, L_{\gamma\delta}] = -i(\eta_{\alpha\gamma} L_{\beta\delta} + \eta_{\beta\delta} L_{\alpha\gamma} - \eta_{\alpha\delta} L_{\beta\gamma} - \eta_{\beta\gamma} L_{\alpha\delta}), \quad (35)$$

$$[T_{rl}, T_{pm}] = -2(\omega_{lp} T_{mr} + \omega_{rm} T_{lp} - \omega_{rp} T_{lm} - \omega_{lm} T_{pr}), \quad (36)$$

$$[L_{\alpha\beta}, T_{rl}] = 0, \quad (37)$$

$$[Q_i^r, L_{\alpha\beta}] = \frac{1}{2} (\Gamma_{\alpha\beta})_i^j Q_i^r, \quad (38)$$

$$[Q_i^r, T^{lp}] = -2(\omega^{rl} Q_i^p - \omega^{lp} Q_i^r) \quad (39)$$

$$\{Q_i^r, Q_j^l\} = \omega^{rl} (\Gamma^{\alpha\beta} C^{-1})_{ij} L_{\alpha\beta} + (C^{-1})_{ij} T^{rl}. \quad (40)$$

The only unknown quantity in this set is the symmetric matrix  $\omega$ . It is the internal group, which will be considered in the next section. whose properties will determine in following.

**Internal Symmetry Group Structure:** It is assumed that the algebra is closed under antilinear involution  $*$  [1-4]. This can always be achieved by extending the algebra to a bigger one which together with every generator G also contains its conjugate  $G^*$ . We take the generators  $L_{\alpha\beta}$  to be anti-self-conjugate (i.e. antihermitian)

$$(L_{\alpha\beta})^* = -L_{\alpha\beta} \quad (41)$$

The conjugates of  $Q_i^r$  and  $T_a$  can be expressed as

$$(Q_i^r)^* = A_{li}^{rj} Q_j^l, \quad (T_a)^* = B_a^b T_b. \quad (42)$$

The involution  $*$  dose not mix fermionic with bosonic generators, nor internal with space-time ones. The involution property  $(*)^2 = 1$  gives consistency conditions

$$A_{li}^{rj} A_{mj}^{*lk} = \delta_m^r \delta_i^k, \quad B_a^b B_b^{*c} = \delta_a^c. \quad (43)$$

constraints on the constants A and B will be derived from the closure of the superalgebra in the involution.

Taking the conjugate of (38) and using (42), we find

$$A_{li}^{rj} (\Gamma_{\alpha\beta})_j^k = ((\Gamma_{\alpha\beta})_i^j)^* A_{lj}^{rk}. \quad (44)$$

Therefore A can be defined by

$$A_{li}^{rj} = E_i^r D_i^j, \quad \text{i.e.} (Q_i^r)^* = E_i^r D_i^j Q_j^l, \quad (45)$$

and the first of the condition (43) becomes

$$EE^*DD^* = 1. \tag{46}$$

If  $DD^* = -1$ , we see that the consistency condition (46) implies that  $EE^* = -1$ . By the use of the conjugate of the anticommutator  $\{Q, Q\}$ , we find the following reality properties

$$(E\omega)^\dagger = -E\omega, (ET)^\dagger = -ET. \tag{47}$$

In the following, for simplicity, the  $N=2$  supersymmetry is considered. One can choose a basis for the  $Q_\alpha^i$  such that  $E$  is a symplectic metric

$$E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{48}$$

By using the reality properties, we can write:

$$E\omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{pmatrix} = \begin{pmatrix} \omega_{21} & \omega_{22} \\ -\omega_{11} & -\omega_{12} \end{pmatrix}, \tag{49}$$

$$(E\omega)^\dagger = \begin{pmatrix} \omega_{21}^* & -\omega_{11}^* \\ \omega_{22}^* & -\omega_{12}^* \end{pmatrix} = -E\omega = \begin{pmatrix} -\omega_{21} & -\omega_{22} \\ \omega_{11} & \omega_{12} \end{pmatrix}. \tag{50}$$

So, we have the following result  $\omega_{21}^* = -\omega_{21}, \omega_{11}^* = \omega_{22}$ , and  $\omega_{22}^* = \omega_{11}, \omega_{12}^* = -\omega_{12}$ . Consequently, we define  $\omega_{11} = s$ ,  $\omega_{22} = s^*$  and  $\omega_{12} = iH$ ,  $\omega_{21} = -iH^\dagger$  therefore

$$\omega = \begin{pmatrix} s & iH \\ -iH^\dagger & s^* \end{pmatrix}. \tag{51}$$

Also from reality properties  $((ET)^\dagger = -ET)$ , we obtain

$$ET = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} T_{21} & T_{22} \\ -T_{11} & -T_{12} \end{pmatrix}, \tag{52}$$

And

$$(ET)^\dagger = \begin{pmatrix} T_{21}^* & -T_{11}^* \\ T_{22}^* & -T_{12}^* \end{pmatrix} = -ET = \begin{pmatrix} -T_{21} & -T_{22} \\ T_{11} & T_{12} \end{pmatrix}. \tag{53}$$

Therefore, we have  $T_{21}^* = -T_{12}, T_{11}^* = T_{22}$  and  $T_{22}^* = T_{11}, T_{12}^* = -T_{21}$ . consequently, we define  $T_{12} = -iH$ ,  $T_{21} = -iH^*$ ,  $T_{11} = a^\dagger$ ,  $T_{22} = a$ . Then we can write

$$T = \begin{pmatrix} a^\dagger & -iH \\ -iH^* & a \end{pmatrix}. \tag{54}$$

Now by using that  $\omega_{ii}$  is symmetry, we have:

$$\omega_{rl} = \omega_{lr} \Rightarrow \begin{pmatrix} s & iH \\ -iH^\dagger & s^* \end{pmatrix} = \begin{pmatrix} s^\dagger & iH^\dagger \\ -iH & s^{\dagger\dagger} \end{pmatrix}. \tag{55}$$

Therefore, we can say  $S^\dagger = S$ ,  $S$  is symmetry and  $H = H^\dagger$ ,  $H$  is real and hermitian. And also by using that  $T_{ii}$  is antisymmetry, we have

$$T_{rl} = -T_{lr} \Rightarrow \begin{pmatrix} a^\dagger & -iH \\ -iH^* & a \end{pmatrix} = -\begin{pmatrix} a^* & iH^\dagger \\ iH & a \end{pmatrix}. \tag{56}$$

Then we can see  $a^\dagger = -a^*$ ,  $a$  is complex and antisymmetry and  $h$  is real. We take into consideration that  $s=0$  and  $H=1$ ,

$$\omega_{rl} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}. \tag{57}$$

In this case, the internal symmetry group is  $O^*(2)$  which the simplest choice [1-4] is.

### CONCLUSIONS

The formalism of the quantum field in de Sitter universe, in ambient space notation, is very similar to the quantum field formalism in Minkowski space. In this paper we present the  $N=2$  de Sitter supersymmetry algebra in this notation, which is independent of the choice of the coordinate system. The importance of this formalism may be shown further by the consideration of the linear quantum gravity and supergravity in de Sitter space, which lays a firm ground for further study of universe.

### ACKNOWLEDGEMENTS

The authors would like to thank S. Rouhani and M.V.takook for useful discussion and IAU-sari branch.

### REFERENCES

1. Pahlavan, A., S. Rouhani and M.V. Takook, 2005.  $N = 1$  de Sitter supersymmetry algebra, Phys. Let. B., 627: 217-223.

2. Li-Ning, Z. and T.L. Nuovo Cimento, 2002. Graded de sitter space-time, Group and Algebra, 46(2): 123-130.
3. Borowiec, A., J. Lukierski and V.N. Tolstoy, 2005. Jordanian quantum deformations of D=4 anti-de sitter and Poincare superalgebras, Eur. Phys. J.C., 44(1): 139-145.
4. Freund, P.G.o., 1986. Introduction to Super symmetry, Cambridge Monographs on Mathematical Physics, Published Cambridge University Press.