# A Model to Increase Flexibility of Precedence Relations in Project Network 

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#### Abstract

This paper provides a model for time reduction in a project through adding linkage intensity in a project network. The traditional network calculation and symbolization are modified and designed a new method to consider the linkage intensity in a network. Then, by using a multiple regression model, a formula for calculation of time reduction potential in a project network is introduced. It is found that there is a meaningful relationship between time reduction and four independent variables as: project complexity, number of activity, linkage intensity type's proportions and whole time of project activities. Two illustrative examples are presented to show the model application for prediction of the potential of time reduction in a project. Using the model and its approach is very simple, because the model does not need any additional information such as the cost of reducing time in a project.


Key words: Project management. precedence network. multivariate regression.time reduction

## INTRODUCTION

Shortening time of project is one of he major concerns of project management researches and experts. It seems natural because the companies which like to remain in the market as pioneer companies have to introduce new products in short time; otherwise they lose the market to their competitors. Today, therefore the life cycle of product is more and more shorten and competition is intensive. Traditionally, the project activity relations are defined in the framework of a network in order to plan a project. One of the influential factors in project time is how to define the different activity relations in the form of a network. Since the precedence relations are not too precise in the real world, it is possible that they are violated in practice and time overlaps occur. To clarify, assume that activity A and activity B are a part of the network with 10 and 15 days duration respectively and activity $A$ is a predecessor activity of B. If you rapid look at actual and planned schedules of these two activities in their Gant chart in Fig. 1, you will see that occurrence those like what these is in this figure is not too unusual! In real world, the precedence links in a network may be violated depending on the type of activity and some overlap may be occurred. This paper seeks to provide a method by which the project planning team can categorize the project links in a flexible way; for example, project links can be categorized into three types such as weak, medium and strong. It is obvious that violating the strong links are not authorized in


Fig. 1: Typical sample of violation dependency in real world
project scheduling, whilst weak links can be violated by some time overlaps. The potential of reducing the time based on the network form are studied as well in the second section of the paper.

## LITRATURE REVIEW

Reduction of project time is one of key topics in the project planning. In general, the studies on project time reducing can be categorized in three types: 1) activity time reduction by using better estimation of activity duration, 2) activity time reduction through
trade-off between time and cost, 3) activity time reduction through changing the project links.

The most famous method of the first type is the critical chain method (CCM). The CCM developers believe that in estimating the activities duration, a significant part of the estimated times for activities contain a buffer for risk reduction so that the duration times may be separated into to parts as main and buffer. The CCM recommends to share the buffer of each task in the whole project and names it the "project buffer" and then put it at the end of the longest activity path (without resources violating) that is called critical chain, therefore by using this approach, the project duration will be decreased significantly [1].

The second type of the studies is an old problem known as the time-cost trade-off problem, which has been recognized as a particularly difficult combinational problem for over decades. Different solutions methods has been offered such as linear programming, integer or dynamic programming or other heuristic or meta-heuristic methods like genetic algorithm [2]. Some scholars have introduced new version of this problem with triple trade-off as cost, time and quality $[3,4]$.

The third type pays particular attention on the time reduction through changes in the links in the project network. Liberatore et al. [5] have developed a quadratic mixed integer programming model and considered the crashing time and changing, reforming or eliminating the precedence relations for project time reduction in their model [5]. They have provided two approaches for accelerating the completion of projects as increasing the parallel activities through completely or partially removed precedence links. This model develops a liner formula of deadline time-cost problem. Vandenbosch and Clift [6] have suggested a method for changing the project network and converting the process of activities from series to parallel model which requires the removal or changes in precedence links [7]. Krishnan et al. [8] have proposed a model-based approach for determining the best overlap made for consecutive activities [8]. It has been formed with a data combination which is exchanged between
upstream and downstream activities at the start of each repeat and resume of a downstream activity.

If resource allocations are introduced into time reduction problem, a vast domain of research will also be introduced. This goes back to the development of "resource constraint project schedule-RCPS" problem. Time reduction in this paper does not consider the resource allocation, therefore, obviously, this paper does not investigate on this problem and the readers who interested in RCPS can refer to the other papers such as [9, 10].

Since the precedence links inherently lack the degree of intensity, in this paper the emphasis has been put on determining the link intensity and applying it in the network model to reduce the time. Implementing this approach is direct and simple and needs no information for exchange of time and cost.

## NETWORK WITH FLEXIBLE LINKS

To increase network flexibility, the degree of link intensity may also be added to the existing data in the network and be shown in network illustration, for example these links may be clarified by weak, medium and strong links and can be presented by a symbol. Figure 2 shows such symbolization for Activity On Node (AON) presentation. The readers for more information about traditional activity nets and AON presentation could refer to Elmaghraby, [11]. As you can see, such links increase the network flexibility degree so that it has a potential to decrease the project time. Hereafter it is called the network with flexible link (NFL). In such networks, some degree of overlap is allowed dependent on the type of the link. To calculate the degree of overlap equation 1 can be suggested:

$$
\begin{equation*}
\mathrm{OL}_{\mathrm{ij}}=\mathrm{n}_{\mathrm{ij}}-\alpha_{\mathrm{ij}} \min (\mathrm{~d}, \mathrm{~d}) \tag{1}
\end{equation*}
$$

Where $\mathrm{n}_{\mathrm{ij}}$ denotes the time lag between activities i and j and $\alpha_{i j}$ denotes a number between 0 and 1 for determining the degree of overlap resulting from intensity of linkage between i and $\mathrm{j} . \mathrm{d}_{\mathrm{i}}$ and $\mathrm{d}_{\mathrm{j}}$ are


Fig. 2: An example for Activity On Node presentation with flexible links
durations of activity $i$ and $j$. Traditional CPM calculation can be corrected as following:

Forward calculations: This calculation starts from the first node in the network to the end node of project network through following steps:

Step 1: Denote $E s_{i}=0$ for all activities without any predecessors

Step2: The earliest finish time of activity $j$ is determined through equation 2

$$
\begin{equation*}
\mathrm{EF}_{\mathrm{j}}=\mathrm{ES}_{\mathrm{j}}+\mathrm{d}_{\mathrm{j}} \tag{2}
\end{equation*}
$$

Step3: The earliest start time for activity j is denoted by $E S_{j} . E S_{j}$ for activities which end to node $j$ is calculated through equation 3

$$
\begin{equation*}
\mathrm{ES}_{\mathrm{j}}=\operatorname{Max}_{\mathrm{k}}\left\{\mathrm{ES}_{\mathrm{k}}+\mathrm{OL}_{\mathrm{kj}}\right\} \tag{3}
\end{equation*}
$$

$k$ is the index of activities which end to node $j$
Step4: step2 and 3 are repeated frequently so that $E F_{j}$ and $\mathrm{ES}_{\mathrm{j}}$ of all activities are obtained. Finally, since there may be several final nodes, the completion time of project $\left(\mathrm{T}_{\mathrm{F}}\right)$ is obtained for final nodes through equation 4 :

$$
\begin{equation*}
\mathrm{T}_{\mathrm{F}}=\operatorname{MAX}\left\{\mathrm{EF}_{\mathrm{j}}\right\} \tag{4}
\end{equation*}
$$

Backward calculation: This calculation is the opposite of forward calculation from the end of project to the beginning of the network through following steps:

Step1: $\mathrm{LF}_{\mathrm{j}}=\mathrm{T}_{\mathrm{F}}$ is determined for each activity without any subsequent link. $L F_{j}$ is Latest finish of activity j

Step2: The latest time for the start of activity $\mathrm{j}\left(L S_{j}\right)$ is always obtained through equation 5

$$
\begin{equation*}
\mathrm{LS}_{\mathrm{j}}=\mathrm{LF}_{\mathrm{j}}-\mathrm{d}_{\mathrm{j}} \tag{5}
\end{equation*}
$$

Step3: $\mathrm{LF}_{\mathrm{j}}$ for other activities that end to j is calculated through equation 6.

$$
\begin{equation*}
\mathrm{LF}_{\mathrm{j}}=\operatorname{MIN}\left\{\mathrm{LS}_{\mathrm{k}}-\mathrm{OL}_{\mathrm{jk}}+\mathrm{d}_{\mathrm{j}}\right\} \tag{6}
\end{equation*}
$$

k is index of activities which end to node j
Step4: Steps 2 and 3 are repeated until $\mathrm{LF}_{\mathrm{j}}$ and $\mathrm{LS}_{\mathrm{j}}$ for all activities are obtained.

## TIME REDUCTION POTENTIAL IN NETWORK WITH FLEXIBLE LINK

It is used 78 projects which have been randomly generated in order to analyze the potential of time reduction. The number of activities in these projects are $10,15, \ldots, 50$. Duration of activities are selected randomly between 10 and 100 . The type link between activities is assumed to be weak, medium and strong and their overlap coefficient $\left(\alpha_{\mathrm{ij}}\right)$ are $1 / 2,1 / 3$ and 0 respectively, the type link is randomly selected. The following variables for each project are recorded:

- Number of activities
- Project complexity: It is calculated by dividing the number of arcs to the number of activities
- Whole duration of activities
- Number of strong, medium and weak links or the whole $\alpha_{i j}$
- Proportion of each of strong medium and weak links in the projects. The proportion of strong links, for example is measured by dividing the number of strong links to the whole number of links.

Reducing the time of project is considered as a response variable in the regression model. The data set that was used in this study is shown in the appendix 1.

Studying the correlation between project time reduction and the variables whom effected on it: In this section the correlation between response variable (time saving) and the variables stated above are investigated:

H1. Studying the correlation between the project time reduction and the whole project complexity. In order to study the correlation between time reduction and the project complexity, the calculate Pearson correlation coefficient for these two variables were found to be 0.204 . The correlation between the time reduction and the complexity, the following hypothesis can be tested:

$$
\left\{\begin{array}{l}
\mathrm{H}_{0}: \rho=0 \\
\mathrm{H}_{1}: \rho \neq 0
\end{array}\right.
$$

where $\rho$ is the real correlation coefficient given the Table 1, p-value for this test was found to be 0.073 , the $\mathrm{H}_{0}$ is rejected at $1 \%$ level of significance. So with a certainty of $90 \%$, the correlation of project save time and its complexity is significant.

H2. To study the correlation between project time reduction and the whole duration of activities.

Appendix 1: Data set of study for 78 random projects

|  | Number activity | Percent alfa 50 | Percent alfa 33 | Percent alfa 0 | Complexity | Sum alfa | Sum duration | Reduce time |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 15 | 0.341 | 0.482 | 0.176 | 5.700 | 28.03 | 905 | 0.213 |
| 2 | 20 | 0.432 | 0.377 | 0.192 | 7.300 | 49.65 | 1213 | 0.249 |
| 3 | 25 | 0.444 | 0.417 | 0.139 | 1.400 | 12.95 | 1369 | 0.182 |
| 4 | 30 | 0.431 | 0.370 | 0.199 | 6.000 | 61.11 | 1497 | 0.200 |
| 5 | 35 | 0.458 | 0.438 | 0.104 | 1.400 | 17.93 | 1749 | 0.239 |
| 6 | 40 | 0.381 | 0.412 | 0.208 | 13.000 | 169.62 | 2298 | 0.226 |
| 7 | 45 | 0.392 | 0.427 | 0.181 | 14.400 | 217.58 | 2535 | 0.229 |
| 8 | 50 | 0.389 | 0.384 | 0.227 | 7.300 | 117.20 | 2624 | 0.198 |
| 9 | 10 | 0.348 | 0.435 | 0.217 | 2.300 | 7.30 | 695 | 0.218 |
| 10 | 15 | 0.397 | 0.413 | 0.190 | 4.200 | 21.08 | 856 | 0.171 |
| 11 | 20 | 0.333 | 0.515 | 0.152 | 1.700 | 11.11 | 1110 | 0.210 |
| 12 | 25 | 0.392 | 0.408 | 0.200 | 5.000 | 41.33 | 1193 | 0.214 |
| 13 | 30 | 0.374 | 0.433 | 0.193 | 11.400 | 112.84 | 1933 | 0.259 |
| 14 | 35 | 0.363 | 0.447 | 0.189 | 10.900 | 125.10 | 1877 | 0.199 |
| 15 | 40 | 0.364 | 0.413 | 0.223 | 15.600 | 198.31 | 2249 | 0.205 |
| 16 | 45 | 0.403 | 0.344 | 0.253 | 3.400 | 48.49 | 2543 | 0.210 |
| 17 | 50 | 0.418 | 0.357 | 0.225 | 7.300 | 118.90 | 2796 | 0.207 |
| 18 | 10 | 0.371 | 0.429 | 0.200 | 3.500 | 11.45 | 597 | 0.207 |
| 19 | 15 | 0.323 | 0.455 | 0.222 | 6.600 | 30.85 | 742 | 0.185 |
| 20 | 20 | 0.405 | 0.378 | 0.216 | 1.900 | 12.12 | 1109 | 0.201 |
| 21 | 25 | 0.379 | 0.389 | 0.232 | 3.800 | 30.21 | 1367 | 0.225 |
| 22 | 30 | 0.446 | 0.411 | 0.143 | 1.900 | 20.09 | 1631 | 0.243 |
| 23 | 35 | 0.413 | 0.369 | 0.219 | 4.600 | 52.47 | 2014 | 0.192 |
| 24 | 40 | 0.352 | 0.467 | 0.180 | 6.100 | 80.62 | 2172 | 0.242 |
| 25 | 45 | 0.405 | 0.408 | 0.187 | 21.900 | 331.83 | 2736 | 0.253 |
| 26 | 50 | 0.401 | 0.417 | 0.182 | 23.000 | 388.07 | 2781 | 0.242 |
| 27 | 10 | 0.381 | 0.238 | 0.381 | 4.200 | 11.30 | 521 | 0.167 |
| 28 | 15 | 0.328 | 0.209 | 0.463 | 4.500 | 15.62 | 868 | 0.172 |
| 29 | 20 | 0.429 | 0.244 | 0.327 | 8.400 | 49.53 | 1070 | 0.170 |
| 30 | 25 | 0.420 | 0.222 | 0.358 | 3.200 | 22.94 | 1471 | 0.206 |
| 31 | 30 | 0.411 | 0.238 | 0.351 | 5.000 | 42.88 | 1738 | 0.188 |
| 32 | 35 | 0.360 | 0.244 | 0.396 | 5.600 | 51.34 | 1876 | 0.128 |
| 33 | 40 | 0.354 | 0.229 | 0.417 | 6.000 | 60.65 | 2380 | 0.174 |
| 34 | 45 | 0.402 | 0.191 | 0.407 | 14.500 | 172.75 | 2355 | 0.166 |
| 35 | 50 | 0.411 | 0.164 | 0.425 | 5.600 | 72.68 | 2746 | 0.169 |
| 36 | 10 | 0.515 | 0.182 | 0.303 | 3.300 | 10.48 | 517 | 0.164 |
| 37 | 15 | 0.402 | 0.165 | 0.433 | 6.500 | 24.78 | 691 | 0.148 |
| 38 | 20 | 0.333 | 0.292 | 0.375 | 2.400 | 12.62 | 1215 | 0.153 |
| 39 | 25 | 0.429 | 0.209 | 0.362 | 11.500 | 81.30 | 1568 | 0.166 |
| 40 | 30 | 0.422 | 0.199 | 0.379 | 5.400 | 44.56 | 1875 | 0.171 |
| 41 | 35 | 0.309 | 0.265 | 0.426 | 3.900 | 32.88 | 1993 | 0.133 |
| 42 | 40 | 0.384 | 0.175 | 0.441 | 5.300 | 52.71 | 2357 | 0.151 |
| 43 | 45 | 0.433 | 0.163 | 0.404 | 2.300 | 28.11 | 2577 | 0.149 |
| 44 | 50 | 0.377 | 0.170 | 0.453 | 3.200 | 38.91 | 2696 | 0.147 |
| 45 | 10 | 0.237 | 0.158 | 0.605 | 3.800 | 6.48 | 556 | 0.103 |
| 46 | 15 | 0.407 | 0.222 | 0.370 | 1.800 | 7.48 | 750 | 0.141 |
| 47 | 20 | 0.435 | 0.156 | 0.409 | 7.700 | 41.42 | 1127 | 0.197 |


| Appendix 1: Continued |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | ---: | ---: | ---: | ---: | :--- |
| 48 | 25 | 0.428 | 0.187 | 0.385 | 7.500 | 51.55 | 1340 | 0.171 |
| 49 | 30 | 0.370 | 0.212 | 0.418 | 11.000 | 84.10 | 1529 | 0.141 |
| 50 | 35 | 0.414 | 0.234 | 0.351 | 3.200 | 31.58 | 1915 | 0.178 |
| 51 | 40 | 0.387 | 0.218 | 0.395 | 8.700 | 92.58 | 2064 | 0.174 |
| 52 | 45 | 0.410 | 0.203 | 0.387 | 18.900 | 231.59 | 2491 | 0.163 |
| 53 | 50 | 0.394 | 0.227 | 0.380 | 4.300 | 58.67 | 2731 | 0.164 |
| 54 | 15 | 0.161 | 0.419 | 0.419 | 6.200 | 20.37 | 874 | 0.090 |
| 55 | 20 | 0.247 | 0.406 | 0.347 | 8.500 | 43.77 | 986 | 0.170 |
| 56 | 25 | 0.188 | 0.430 | 0.383 | 11.900 | 70.24 | 1485 | 0.141 |
| 57 | 30 | 0.205 | 0.378 | 0.416 | 6.200 | 42.10 | 1728 | 0.155 |
| 58 | 35 | 0.153 | 0.470 | 0.377 | 6.100 | 49.83 | 1936 | 0.137 |
| 59 | 40 | 0.154 | 0.438 | 0.408 | 3.300 | 28.81 | 1950 | 0.113 |
| 60 | 45 | 0.200 | 0.400 | 0.400 | 3.000 | 31.32 | 2420 | 0.133 |
| 61 | 50 | 0.208 | 0.377 | 0.415 | 4.100 | 47.24 | 2680 | 0.130 |
| 62 | 10 | 0.194 | 0.528 | 0.278 | 3.600 | 9.77 | 480 | 0.175 |
| 63 | 20 | 0.240 | 0.404 | 0.356 | 5.200 | 26.36 | 1103 | 0.164 |
| 64 | 25 | 0.202 | 0.413 | 0.384 | 9.700 | 57.50 | 1177 | 0.160 |
| 65 | 30 | 0.214 | 0.358 | 0.428 | 7.600 | 51.56 | 1915 | 0.178 |
| 66 | 35 | 0.224 | 0.373 | 0.403 | 8.400 | 69.30 | 1977 | 0.173 |
| 67 | 40 | 0.180 | 0.391 | 0.429 | 17.500 | 153.42 | 2175 | 0.152 |
| 68 | 45 | 0.181 | 0.448 | 0.371 | 7.800 | 84.14 | 2639 | 0.148 |
| 69 | 50 | 0.202 | 0.485 | 0.313 | 3.300 | 42.57 | 2730 | 0.161 |
| 70 | 10 | 0.077 | 0.615 | 0.308 | 1.300 | 3.14 | 560 | 0.154 |
| 71 | 15 | 0.000 | 0.522 | 0.478 | 1.500 | 3.96 | 707 | 0.121 |
| 72 | 20 | 0.180 | 0.451 | 0.369 | 6.100 | 29.15 | 1229 | 0.154 |
| 73 | 25 | 0.154 | 0.484 | 0.363 | 3.600 | 21.52 | 1393 | 0.167 |
| 74 | 30 | 0.250 | 0.357 | 0.393 | 3.700 | 27.20 | 1671 | 0.148 |
| 75 | 35 | 0.226 | 0.368 | 0.406 | 16.500 | 134.96 | 2000 | 0.142 |
| 76 | 40 | 0.206 | 0.373 | 0.421 | 6.300 | 57.02 | 2031 | 0.127 |
| 77 | 45 | 0.188 | 0.444 | 0.367 | 11.600 | 125.23 | 2112 | 0.146 |
| 78 | 50 | 0.220 | 0.423 | 0.357 | 10.300 | 128.11 | 2838 | 0.173 |
|  |  |  |  |  |  |  |  |  |

Table 1: Pearson correlation coefficient

|  |  | Reduce time | Complexity |
| :--- | :--- | :---: | :---: |
| Reduce time | Pearson correlation | 1 | 0.204 |
|  | Sig. (2-tailed) |  | 0.073 |
|  | N | 78 | 78 |
| Complexity | Pearson correlation | 0.204 | 1 |
|  | Sig. (2-tailed) | 0.073 |  |
|  | N | 78 | 78 |

Table 2: Pearson correlation coefficient

|  |  | Reduce time | Sum duration |
| :--- | :--- | :--- | :--- |
| Reduce time | Pearson correlation | 1 | 0.103 |
|  | Sig. (2-tailed) |  | 0.371 |
|  | N | 78 | 78 |
| Sum duration | Pearson correlation | 0.103 | 1 |
|  | Sig. (2-tailed) | 0.371 |  |
|  | N | 78 | 78 |

The correlation coefficient between the project time reduction and the whole duration of activities was found to be 0.103 and $\mathrm{p}=0.371$ (Table 2). Therefore, it can be concluded that there is no significant correlation between project time reduction and the whole duration of activities.

H3. To study the correlation between the time reduction and the total $\alpha_{\mathrm{ij}}$

The correlation coefficient between the time reduction and the total $\alpha_{i j}$ on all arcs in the network of project was found to be 0.343 and $\mathrm{p}=0.02$ (Table 3). In other words, there is a positive correlation between project time reduction and the summation $\alpha_{i j}$ on all arcs in the network of project.

H4. To Study the correlation between the time reduction and the percentage of weak, medium and strong links.

The correlation coefficients between the time reduction and the proportion of weak, medium and


Fig. 3: Scatter plot of independent variables vs. response variable
strong links found to be $0.557,0.231$ and- 0.824 respectively (Table 4). The test of significance for correlation for each of these two variables is rejected at $5 \%$ significance level. In other words there is a positive and strong relation between the project time reduction and the weak links. Increasing proportion strong links
results in less time saving and increasing the weak links results in the more time saving.

The following diagrams (Fig. 3) show the relation between the time reductions, complexity, whole duration of activities, the summation $\alpha_{i}$ and the number of activities in the form of points.


Fig. 4: Normal Q-Q Plot of unstandardized residual

Table 3: Pearson correlation coefficient

|  |  | Reduce time | Sum alpa |
| :--- | :--- | :---: | :---: |
| Reduce time | Pearson correlation | 1 | $0.343^{* *}$ |
|  | Sig. (2-tailed) |  | 0.002 |
|  | N | 78 | 78 |
| Sum alpa | Pearson correlation | $0.343^{* *}$ | 1 |
|  | Sig. (2-tailed) | 0.002 |  |
|  | N | 78 | 78 |
| **Correlation is significant at the 0.01 level (2-tailed) |  |  |  |

Proposed model: A multivariate regression model is used for predicting the project time reduction. Readers for more information about multivariate regression can refer to related references such as [2]. The variables affecting the project time reduction are selected. As soon as the model was implemented, the effect of each variable is statistically tested in the model. In the model $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}, \mathrm{X}_{4} \mathrm{X}_{5}$ are independent variables and Y is the dependent variable.



Fig. 5: Observed reduce time and predicted reduce time

$$
Y=\text { Intercept }+\beta_{1} X_{1}+\beta_{2} X_{2}+\beta_{3} X_{3}+\beta_{4} X_{4}+\beta_{5} X_{5}+\varepsilon
$$

Where $\varepsilon$ is random error and $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$ are the number of activities, project complexity, the whole durations of activities, the proportion of the number of weak links and the proportion of the number of medium links respectively and $Y$ is the project time reduction.

The hypotheses of zero regression coefficients are:

$$
\text { H5. }\left\{\begin{array}{l}
H_{0}: \beta_{i}=0 \\
H_{1}: \beta_{i} \neq 0 \quad i=1,2, \ldots, 5
\end{array}\right.
$$

The first column in Table 5 indicates the variables which are in the regression model. The columns 2 and 3 indicate the non-standardized coefficient of variables and the standard errors of the estimated regression coefficients respectively. The column s 4 and 5 show the standardized coefficients and T-student statistic for testing H5 respectively.

The last column shows P -value for testing. In a confidence level of 0.95 , all variables are significantly put into the model. Also $\mathrm{R}^{2}$ and the adjusted- $\mathrm{R}^{2}$ of the resulted model are $0.991,0.990$ respectively.

Testing the significance of the estimated regression model: It should be noted that if all the variables of the regression model could be equaled to zero significantly. it is as follows:

$$
\text { H6. }\left\{\begin{array}{l}
\mathrm{H}_{0}: \beta_{\mathrm{i}}=0 \quad \text { for } \forall \mathrm{i} \\
\mathrm{H}_{1}: \beta_{\mathrm{i}} \neq 0 \text { foratleastonei }
\end{array}\right.
$$

The regression model was found to be highly significant ( $\mathrm{p} \ll 0.0$ ).

Validity of the model: In order to check the validity of the regression model, the normality of the residuals were tested. The following table shows Shapiro-Wilk's statistic for the normality of the residuals ( $p=0.517$ ). The quantity of this statistics is 0.985 and p -value for the test is 0.517 (Table 7). So at $5 \%$ level of significant, there is no reason for rejecting the normalization of the residuals. The following diagram shows the Normal Probability Plot.

Using the model for prediction: Another criterion for models suitability is the low mean squares of error (MSE) in regression model.

$$
\mathrm{MSE}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\mathrm{Y}_{\mathrm{i}}-\hat{Y}_{\mathrm{i}}\right)^{2}}{\mathrm{n}-\mathrm{k}}
$$

Where n and k are the number of observations and the number of independent variables respectively in the model. The ANOVA table (Table 6) shows MSE of the estimated model. Figure 5 shows the actual reduced

Table 4: Pearson correlation coefficients

|  |  | Reduce time | Percent alpa 50 | Percent alpa 33 | Percent alpa 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Reduce time | Pearson correlation | 1 | 0.557** | 0.231* | -0.824** |
|  | Sig. (2-tailed) |  | 0.000 | 0.041 | 0.000 |
|  | N | 78 | 78 | 78 | 78 |
| Percent alpa 50 | Pearson correlation | 0.557** | 1 | 0.552** | -0.431** |
|  | Sig. (2-tailed) | 0.000 |  | 0.000 | 0.000 |
|  | N | 78 | 78 | 78 | 78 |
| Percent alpa 33 | Pearson correlation | 0.231* | -0.552** | 1 | -0.514** |
|  | Sig. (2-tailed) | 0.041 | 0.000 |  | 0.000 |
|  | N | 78 | 78 | 78 | 78 |
| Percent alpa 0 | Pearson correlation | -0.824** | -0.431** | -0.514** | 1 |
|  | Sig. (2-tailed) | 0.000 | 0.000 | 0.000 |  |
|  | N | 78 | 78 | 78 | 78 |

$\overline{* *}$ Correlation is significant at the 0.01 level (2-tailed), $*$ Correlation is significant at the 0.05 level (2-tailed)

Table 5: Regression results

| Model |  | Unstandardized coefficients |  | Standardized coefficients |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | B | Std. Error | Beta | t | Sig. |
| 1 | Number activity | -0.003 | 0.001 | -0.572 | -3.169 | 0.002 |
|  | Complexity | 0.001 | 0.000 | 0.044 | 2.014 | 0.048 |
|  | Sum duration | $5.54 \mathrm{E}-0.005$ | 0.000 | 0.568 | 3.108 | 0.003 |
|  | Percent alfa 50 | 0.291 | 0.014 | 0.557 | 20.967 | 0.000 |
|  | Percent alfa 33 | 0.217 | 0.012 | 0.442 | 17.746 | 0.000 |

Dependient variable: reduce time

Table 6: ANOVA analysis

| Model |  | Sum of squares | df | Mean square | F | Sig. |
| :--- | :--- | :---: | :--- | :---: | :---: | :---: | :---: |
| 1 | Regression | 2.507 | 5 | 0.501 | 1027.796 | $0.000^{\mathrm{a}}$ |
|  | Residual | 0.037 | 76 | 0.000 |  |  |
|  | Total | 2.544 b | 81 |  |  |  |

Predictors: Percent alfa 33, Complexity, Percent alfa 50 Number activity, Sum duration and Dependent variables: Reduce time

Table 7: Test of normality

|  | Kolmogorov-smirnov ${ }^{\text {a }}$ |  |  | Shapiro-wilk |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Statistic | df | Sig. | Statistic | df | Sig. |
| Unstandardized residual | 0.059 | 78 | 0.200* | 0.985 | 78 | 0.517 |

*This is a lower bound of the true significance, ${ }^{\text {a }}$ Liliefos significance correction

Table 8: Duration of activity

| 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | Activity |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 56 | 33 | 39 | 46 | 72 | 48 | 18 | 84 | 5 | 25 | Duration |

Table 9:

| Number of <br> Activity | Percent <br> alfa.50 | Percent <br> alfa.33 | Complexity | Sum <br> duration | TF1 | Reduce <br> Time2 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Network1 | 10 | 0.50 | 0.33 | 1.2 | 426 | 316 | 248.06 |



Fig. 6: Network 1
time in the projects compared to the predicted value for time reducing in 78 projects.

Example 1: Consider the following network1: (Fig. 6).
The relations between the activities are categorized as weak, medium and strong. They are shown with three different arrows. The value of a in these relations are $1 / 2,1 / 3$ and zero respectively.

The following tables (Table 8 and 9) show the summary information for the above network. The
number of activities in the network is 10 . The proportions of weak, medium and strong links are $0.5,0.33$ and 0.17 respectively and the network complexity is 1.2 . With out considering the three links of flexibility, the time of completed project is 316 time units. The links of flexibility, it is 248.6 time units. So the proportion of the saved time to the time finish (TF1) is 0.215 and the proportion of the saved time predicted by the regression model is 0.2305 .

Table 10: Duration of activity
$\left.\begin{array}{llllllllllllll}\hline 15 & 14 & 13 & 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2\end{array}\right] 1$ Activity

Table 11:

|  | Number of <br> Activity | Percent <br> alfa 50 | Percent <br> alfa33 | Complexity | Sum <br> duration | TF1 | TF2 | Reduce <br> Time |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Network2 | 15 | 0.4 | 0.32 | 1.67 | 927 | 469 | 373.37 | 0.2039 |



Fig. 7: Network 2

Example 2: Consider the network2: (Fig. 7)
The following tables show the summary information for the above network (Table 10, 11). The number of activities in this network is 15 . The proportions of weak, medium and strong links are 0.4 , 0.32 and 0.28 respectively and the network complexity is 1.67. With out considering the three links of flexibility, the time of completed project is 469 time units and the links of flexibility, it is 373.37 time units.

So the proportion of the saved time to the time finish (TF1) is 0.2039 and the proportion of the saved time predicted by the regression model is 0.2336 .

## CONCLUSION

This article addresses the project time reduction through ranking the network links. Since the intensity of the links cannot be defined in conventional networks, the flexible networks are defined and the methods of calculation are introduced. The results show that the
project time reduction has positive and significant relation to network complexity, the summation of overlap coefficient $\left(\alpha_{\mathrm{ij}}\right)$ on all arcs in the network of project, the proportion of weak and medium links but it doesn't have a significant relation to the whole time of activities.

To determine the potential of time reduction in the network, a multiple linear regression model was used. The model variables were the number of activities, project complexity, the whole time for doing the project activities and the proportion of weak and medium links.

The $R^{2}$ was found to be 0.991 which shows that regression variables explain the changes in project time reduction at a rate of 0.991 . Validity of the regression model and its assumption were tested (Significant $\mathrm{R}^{2}$, Normality of the residuals and the low MSE). Two examples were used to show that how the regression model can predict the potential for time reduction. This paper suggests the three links of weak, medium, strong; and certain values were introduced for the overlapping
amount. This approach is simple and direct and can be useful to decrease project time without any cost information and also for calculation of the potential of time reduction in a defined project. Further study on calculation system by using advanced method such as fuzzy linguistic variables is recommended to improve the results.

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