

## Design of Low Complexity Polar Coded Spatial Modulation

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**Abstract:** Spatial Modulation has emerged as the new Multi Input Multi Output (MIMO) scheme offers better bandwidth efficiency. The specialty of Spatial Modulation is that only one antenna is active during a symbol period while the rest of the antennas are in idle mode. This feature eliminates the issues related to Inter Channel Interference (ICI) and Inter Antenna Synchronization (IAS). Several schemes have been developed and all are require an antenna index detection algorithm at the receiver. The key idea of this project is to solve the antenna detection problem and reduce the complexity of the receiver. Polar coded spatial modulation (PCSM) is proposed to provide both detection/decoding of the transmitted symbol and transmit antenna index in multiple-input-multiple-output (MIMO) antenna system. Polar coding, recently invented by Erdal Arıkan, is an encoding/decoding scheme that provably achieves the capacity of the class of symmetric binary memoryless channels. In polar coded system, free polar set is used for mapping transmitted information and frozen polar set for transmit antenna index. Consequently, the bit error rate performances of 8 phase shift keying modulation SM are analysed for various MIMO architecture and for low complexity results, ML detection are used at the receiver. This detection process is only for free polar set.

**Key words:** Frozen Polar set • Free polar set • MIMO • Polar code • Spatial Modulation

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### INTRODUCTION

Multiple-input-multiple-output (MIMO) wireless systems are those that have multiple antenna elements at both the transmitter and receiver. The multiple antennas in MIMO system can be exploited in two different ways. Firstly, the creation of a highly effective antenna diversity system; the other is the use of the multiple antennas for the transmission of several parallel data streams to increase the capacity of the system. Spatial Multiplexing is a novel technique which allows all the antennae to be active at the transmission part, which in turn requires inter-antenna synchronization (IAS) for simultaneous transmission from all the antennae and also this method suffer from inter-channel interference (ICI) [1]. To overcome these problems a method is suggested so called Spatial Modulation (SM) [2]. This is the new method emerged in the MIMO scheme that provide high bandwidth efficiency. SM dissolves the problem of ICI and IAS. The uniqueness property of SM is that only one

antenna is active during the whole transmission and rest of the antennas remains inactive during the transmission. Another main feature of SM is that, unique detection of each antenna number that is possible at the receiver during the transmission, which is called as antenna index [3].

Depending upon the SM scheme, several novel SM schemes have been developed. One among such schemes is Trellis code Spatial Modulation (SM), this idea is applied to antenna constellation point. Whereas the entire above scheme requires detection algorithm at the receiver for detecting antenna number [4]. To overcome the complexity in detection algorithm and problem in antenna number determination, a new method in coding scheme is implemented. This method relays its idea on polar codes and it forms the main contribution of this work. In this method polar codes are divided into two sets: 1. free polar set and 2. Frozen polar set. Free polar set are nothing but the transmitting antenna symbols, the bits in the frozen polar set position are normally fixed and this information

is known at both transmitting and receiving end. In this paper, the main idea of polarization technique of polar codes is applied to identify the transmit antenna for spatial modulation. Since the transmit antenna information is relayed to the receiver without sending those bits that identify the transmit antenna. Then the received data symbols are then simply demapped and combined with the correct antenna indices whose values and position are already known at the receiver. This eliminates the antenna number detection overhead at the receiver.

In this proposed work, 8 phase shift keying modulation scheme is used to modulate the symbol bits, whereas the bits that identify the transmit antenna index are a preset random sequence encoded into frozen polar set and this sequence is also known at the receiver. Therefore, there is no need of transmitting antenna index to the receiver. However, the performances of proposed system are analyzed in the presences of Rayleigh flat fading and Rician channels. The analysis is based on the channel capacity results derived for various QPSK modulation schemes with natural mapping.

**Notation:** In this proposed method,  $M$  denotes the size of the complex signal constellation,  $E_b$  is the transmitted signal energy per source bit.  $W^N$  denotes the operation modulo-2 addition.  $W^N$  denotes the channel corresponding to  $N$  uses of channel  $W$  while  $W(y|x)$  denotes the transition probability over the channel when output  $y$  is observed given that the encoded input  $x$  is transmitted. The  $i^{\text{th}}$  synthetic channel is denoted by  $W_N^i$ . In addition, It is used  $|\cdot|, \|\cdot\|_F, (\cdot)^H, E[\cdot]$  and  $[\cdot]^T$  to denote the magnitude, Frobenius norm, Hermitian, expectation and the transpose of a vector or matrix, respectively. Bold italics lower/upper case symbols denote vectors/matrices, whereas regular letters represent scalar quantities.

**Polar Code:** Polar coding, recently invented by Erdal Arkan [5], is an encoding/decoding scheme that provably achieves the capacity of the class of symmetric binary memoryless channels. In particular, the algorithm can find almost all the “good” channels with computing complexity which is essentially linear in block-length. Different methods and algorithms were used to enhance the performance of the successive cancellation polar decoder. This paper provides numerical evidence as well as some mathematical analysis to show that the performance of polar codes is improved by using these algorithms. The polar codes are constructed such that two sets of channel are differentiated: free polar set and frozen polar set. This method of construction is known as channel polarization.

**Channel Polarization:** Channel polarization is an operation which produces  $N$  channels  $\{W_N^{(i)}: 1= i = N\}$  from  $N$  independent copies of a B-DMC  $W$  such that the new parallel channels are polarized in the sense that their mutual information is either close to 0 (completely noisy channels) or close to 1 (perfectly noiseless channels) [5].

**Channel Parameter:** The two channel parameters of interest are the symmetric capacity.

$$I(W) \square \sum_{y \in Y} \sum_{x \in X} \frac{1}{2} W \left( \frac{y}{x} \right) \log \frac{W \left( \frac{y}{x} \right)}{\frac{1}{2} W \left( \frac{y}{0} \right) + \frac{1}{2} W \left( \frac{y}{1} \right)} \quad (1)$$

and the Bhattacharyya parameter

$$Z(W) \square \sum_{y \in Y} \sqrt{W(y/0)W(y/1)} \quad (2)$$

These parameters are the measures of rate and reliability, respectively.

**Free Polar Set and Frozen Polar Set:** Polar code rate is given by  $R$  and size of free polar set is given by  $K=RN$ . The encoded free polar set is given as  $X_f \subseteq (1,2, \dots \dots \dots K)$  while the set of frozen bits is given as  $X^f \subseteq [(k+1), \dots \dots \dots, N]$ . Where  $u_f$  and  $u_e$  denote the source bit vector mapped into the free and frozen sets, respectively. Consequently, the encoded output sequence is in the following way,  $x_i^N = u_f G_N(X_f) \oplus u_e G_N(X_e)$ , where  $G_N(X_f)$  denotes the sub-matrix of  $G_N$  formed by the rows with the indices in  $X_f$  and  $G_N(X_e)$  denotes the sub-matrix of  $G_N$  formed by rows with the indices in  $X_e$ . Hence, polar codes are identified by the parameter vector  $(N, K, X_f, X_e)$  and since  $K$  and  $N$  are fixed, the next step is to determine the sets  $X_f$  and  $X_e$ . For Example, given.

$$G_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, x_1^4 = u_1^4 G_4 \\ = (u_2, u_4) \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} + (1,0) \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

As the encoder mapping for  $(4, 2\{2, 4\}, (1, 0))$  code. This means that for a source block  $(u_2, u_4) = (1,1)$  the coded block is  $x_1^4 = (1,0,1,0)$ .

Consequently, from the symmetric channel capacity  $I(W)$ , the indices in the information/free set  $X_f$  are chosen as a  $K$ -element subset of  $\{1, 2, 3 \dots N\}$  such that  $I(W_N^{(i)} \geq I(W_N^{(j)})$  for all  $i \in X_f$  and  $j \in X_e$  and this forms the basis for polar coding. Therefore the channel capacity (transmission rate) is a vital requirement for the code

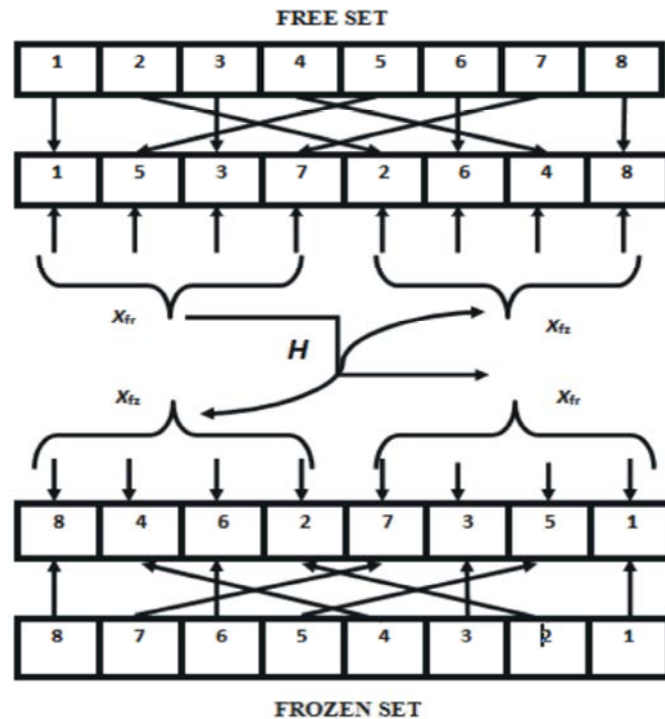


Fig. 1: Polar coded mapping strategy for spatial modulation

construction. The bits in the free polar set (odd indices) are transmitted at rate one through the channel  $H$  while the bits in the frozen set (even indices) are fixed, known at the receiver and the indices are used to convey the transmit antenna index. This novelty in polar coding that has been adopted to use in PCSM scheme. The following section describes the system model for PCSM [6].

**Polar Coded Sm System Model**

**Encoding:** The proposed model consists of  $N_t \times N_r$  PCSM communication system, where  $N_t$  is the transmitting antenna and  $N_r$  is the receiving antenna, respectively. Binary source input bits is given by  $u$ . And size of  $u$  is denoted by  $b$ , where it is given by a formula  $b = \log_2(MN_t)$  [7, 8], The sources bit is  $u$  are encoded into the output vector  $X$  which is then separated into two binary encoded output sets,  $X_{fr}$  and  $X_{fz}$  according to the code rate  $R$  where  $X_{fr}$  denotes the set of bits mapped into free polar set, whereas  $X_{fz}$  denotes the set of spatial bits mapped into frozen polar set [9].

The spatial mapper assigns  $X_{fz}$  to a transmit antenna index vector  $t$  while the symbol mapper assign  $X_{fr}$  to an 8psk symbol  $X_m$ ,  $m \in (1:M)$ . After code mapping, the symbol vector is represented as,  $a_m(k) = [a_m(1), \dots, a_m(S)]$ , where  $S = \lfloor F_{tot} / \log_2(M) \rfloor$  and  $co(1:S)$ . Herein,  $a_m$  denotes the complex signal with  $E[a_m(k)^2] = 1$ , where  $E[\cdot]$  denotes the expectation operator. Also, in a single frame, only the

subset of  $t$  which satisfies the following is used:  $t(k) = [t_j(1), \dots, t_j \left( R_m \times \left( \frac{[N-K]}{\log_2(N_t)} \right) \right)]$ ,  $j \in (1:N_t)$ ,  $R_m$  refer to the ratio

of the actual number of bits used to assign the antenna indices to the number of the bits in the whole frozen polar set. Actually,  $\left( \frac{[N-K]}{\log_2(N_t)} \right) = S$ . The assignment of the vector

$t$  and  $x_m$  are defined by SM mapping table are used. This is known at both transmitter and receiver. At this point, it is imperative to determine the number of transmit antenna that can be accommodated by a given polar code frame after the symbol are modulated by the conventional MPSK schemes.

**Transmission:** On allocating the symbols in the code frame, the polar coded SM mapper then assign the symbol to the active transmit antenna which follows the SM mapping. For each symbol period, the  $S$  symbols  $x_{mj}(k) = [x_m(k), x_m(k+1), \dots, x_m(k+N_t-1)]^T$ ,  $j \in \{1:N_t\}$ , which choose  $N_a \leq N_t$  to denote the number of active transmit antenna is active for the  $k^{th}$  symbol transmission, only one element of  $x_{mj}(k)$  will be a non-zero element. Since the  $(N_a \times 1)$  signal vector output of the SM mapper associated with antenna  $t_j$  is written as  $x_{mj}(k) = [00 \dots x_m(k) \dots 0]^T$  where only the symbol at the  $j^{th}$  position corresponding to the active antenna is transmitted. Consequently, all the  $S$  symbols in the frame are then successively transmitted

from a single antenna over the  $N_r \times N_t$  MIMO channel  $H$ . In this proposed work,  $H$  is modeled as an  $N_r \times N_t$  complex MIMO channel matrix with the element representing the path gains  $h_{ij}$  between the receive antenna  $t_i$  and transmit antenna  $t_j$ , where  $i \in \{1:N_r\}$  and  $j \in \{1: N_t\}$ . This channel model may be written as,

$$H = \begin{pmatrix} h_{11} & \cdots & h_{1N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r1} & \cdots & H_{N_rN_t} \end{pmatrix} \quad (3)$$

There are two channel model are evaluated in this paper. In first case, Rayleigh distribution for non-line of sight (NLOS) application is used. On the other hand Rician distribution is used for line of sight (LOS).

**Decoding:** The perfect channel state information is available at the receiver, where  $N_r \times N_t$  channel responses are required. For the PCSM system, the frozen polar set position in the codeword are known at the receiver, therefore the preset sequence of the bits that identify the antenna index are used in the frozen positions to identify the respective transmit antenna for the received symbols while utilizing the mapping table. OFDM is a block modulation scheme where a block of  $N$  information symbols is transmitted in parallel on  $N$  subcarriers. The time duration of an OFDM symbol is increased by  $N$  times than that of a single carrier system. An OFDM modulator can be implemented using an inverse discrete Fourier transform (IDFT) on a block of  $N$  information symbols followed by an analog to digital converter (ADC). To mitigate the effects of inter symbol interference (ISI) on the  $N$ -IDFT coefficients caused by channel time spread, each block is preceded by a cyclic prefix (CP) or a guard interval consisting of  $L$  samples, such that the length of the CP is at least equal to the channel length,  $(N+L)$ . Thus the linear convolution between the transmitted sequence and the channel is converted to a circular convolution. Therefore the effects of the ISI are completely eliminated easily. The use of fast signal processing transforms such as a fast Fourier transform (FFT) for OFDM implementation enables. This need to use frequency domain equalization at the receiver while using similar techniques in single carrier systems, by preceding each transmitted data block of length  $N$  by a cyclic prefix of length  $L$ .

The technique of arranging multiple antennas at both the transmitter and receiver is called Multiple-Input Multiple-Output (MIMO) system. In a MIMO system, spatial diversity is obtained by spatially separated antennas in a dense multipath scattering environment.

MIMO systems are implemented in a number of different ways to overcome signal fading using diversity gain, or to obtain a capacity gain. The MIMO techniques aims to improve the power efficiency by maximizing spatial diversity using Space-time block codes.

**MIMO-OFDM System Model:** For single antenna system, i.e., SISO, the transmitted signal is represented by  $s(k)$  and  $N$ -point IFFT is used in the transmitter section. The data samples are converted from serial to parallel streams after encoding and interleaving in order to reduce random and burst noise on the data signals. Then  $N$ -point IFFT is done on the data samples after some digital modulation schemes like Binary Phase Shift Keying (BPSK), Quadrature Amplitude Modulation (QAM) etc., After IFFT, the last  $L$  data samples are added to the  $N$  data samples as cyclic prefix (CP) to generate one OFDM symbol  $s_n$  of length  $N+L$ . Frames are built in the next step and sent to channel. Then the received signal, along with its frequency offset is expressed in the presence of Additive White Gaussian Noise (AWGN).

$$r(k) = s(k) \exp\left(\frac{j2\pi k\epsilon}{N}\right) + w(k) \quad (4)$$

For multiple antenna system, i.e., MIMO, with  $Q$  transmit antennas and  $L$  receive antennas,  $s_i(k)$  is the transmitted signal from the  $i^{\text{th}}$  transmit antenna, then  $r_l(k)$  which is the received signal on the first receive antenna is given by,

$$r_l(k) = \sum_{i=1}^Q h_{i,l} s_i(k) \exp\left(\frac{j2\pi k\epsilon}{N}\right) + w_l(k) \quad (5)$$

where  $w_l(k)$  is the additive white Gaussian noise (AWGN) on the first receive antenna with zero mean and variance  $\sigma_l^2$ ,  $h_{i,l}$  is the channel gain between the  $i^{\text{th}}$  transmit antenna and the first receive antenna and  $\epsilon$  denotes the CFO normalized to the inter-carrier spacing. The CFO estimation includes: acquisition and tracking. A large range of CFO can be estimated in acquisition, whereas the estimation range is much smaller in tracking. But, the estimation accuracy of tracking is better than that of acquisition. After acquisition, the remaining CFO must be within the tracking range for the tracking algorithm to be precise.

**ML Estimator with Proposed Block:** The MLE algorithm is adopted as part of the frequency correction scheme, as it is a highly efficient non-data aided synchronization method and can be implemented with a common block

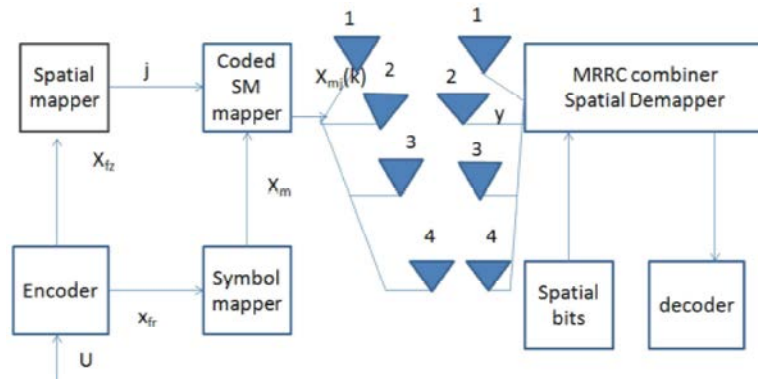


Fig. 2: Proposed PCSM System Model

using the frequency tracking algorithm, which is shown in Figure 2. The left half of the figure stands for the process of MLE algorithm and the remaining part takes care of the frequency tracking.

The ML estimation for  $\theta$  and  $\epsilon$  lies in maximizing the argument  $\Lambda(\theta, \epsilon)$  and under the assumption that  $r$  is a jointly Gaussian vector and the log-likelihood function.

$$\Lambda(\theta, \epsilon) = |\gamma(\theta)| \cos(2\pi\epsilon + \angle\gamma(\theta)) - \rho\Phi(\theta) \quad (6)$$

$$\rho\Delta \left| \frac{E\{r(k)r^*(k+N)\}}{\sqrt{E\{|r(k)|^2\}E\{|r(k+N)|^2\}}} \right| \quad (7)$$

where  $\rho$  is the magnitude of the correlation coefficient between  $r(k)$  and  $r(k+N)$ .

$$\rho = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2} = \frac{SNR}{SNR + 1} \quad (8)$$

### RESULT

The simulation of PCSM is carried out in the matlab environment. The simulation scenario uses 4X4 MIMO system with BEC. The performance of PCSM is analyzed for different modulation schemes like QPSK, 8-PSK, 16-PSK, 4-PSK. In Fig. 3, Polar coded performances are shown for various MIMO channel with 4X4, 8X8, 16X16.

Table 1: SNR Vs BER

SNR	BER
0	4.978027e-01
2	4.992676e-01
4	2.288818e-01
6	7.695312e-02
8	1.445312e-02
10	1.782227e-03
12	1.098633e-04
14	1.525879e-05

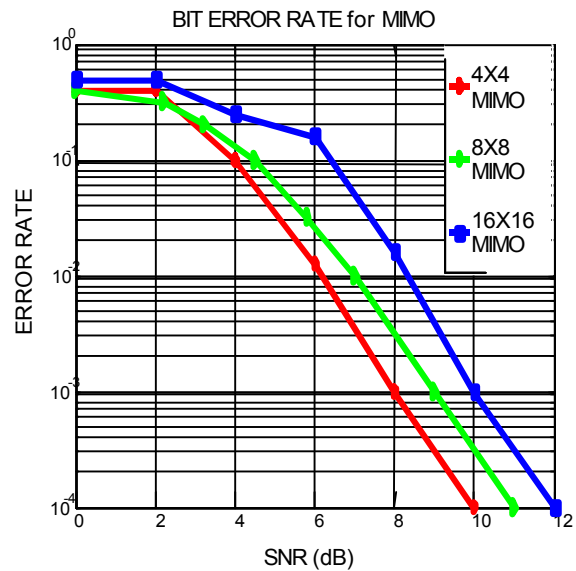


Fig. 3: Bit Error rate performances for various MIMO channel

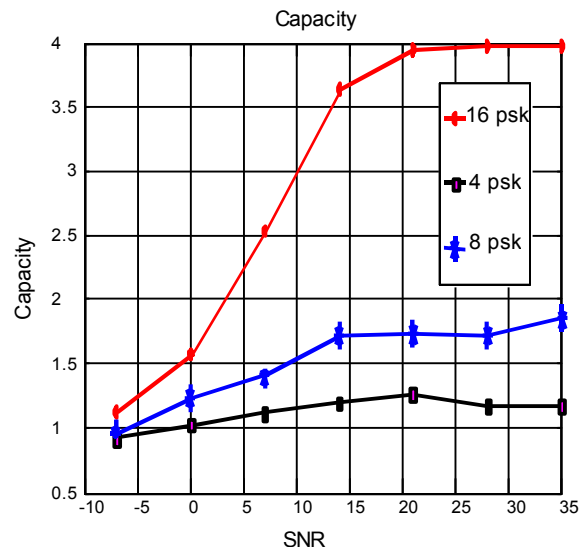


Fig. 4: Capacity result for various Qpsk

In Fig. 4, the capacity result for various Qpsk is shown with  $Q = 4, 8, 16$ . The graphs for all the three schemes exhibit a normal error flow. The above shown table1 give the SNR versus Bit Error Rate value. The Simulation results show that the performance of PCSM is better than the conventional MIMO schemes.

### CONCLUSION

In this paper, the concept of polar codes are studied in detail to relieve the transmit antenna detection problem which is used in the conventional SM. The novelty of the code is that it uses the frozen polar set which is identifiable at the receiver, whose position are used at the receiver to map the bits that identify the transmit antenna. Hence the spectral efficiency is enhanced. Also it reduces the receiver complexity. The performance of this scheme may be improved in future by incorporating with the smart antenna system.

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