Middle-East Journal of Scientific Research 22 (4): 621-625, 2014
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2014.22.04.21937

# Designing and Construction of Group Sampling Plans for an M/M/C Queueing Model

*[1]S. Murali, [2]K.S. Ramaswami and [3]S. Devaarul*

[1]Department of Mathematics, Jansons Institute of Technology, Coimbatore-641 659, Tamil Nadu, India
[2]Department of Mathematics, Coimbatore Institute of Technology, Coimbatore-641 014, Tamil Nadu, India
[3]Department of Statistics, Government Arts College, Coimbatore-641 018, India

**Abstract:** In managing a production process with queueing system, the important and desirable event is to determine the expected number of Quality control inspectors Servers(C). Once c is known, the next is to determine the parameters of the sampling plans for testing purpose. In this article Group sampling plans are developed for a M/M/C production process. A novel algorithm is given for sentencing the group of batches or lots which comprises of components. The sampling plans are developed based on the expected number of arrivals in the system. The quality domain, quality and efficiency domain are derived and provided for the queueing control system. Tables are constructed to select the number of quality control inspectors (servers) and to select the parameters of the sampling plans.

**Key words:** Group Sampling Plans · M/M/C Production process · Traffic intensity · Expected number of arrivals

## INTRODUCTION

A Queueing system comprises of batches or lots arriving for service, waiting for service and then leaving the system after service. In Quality Control literature, the group of components arrive for testing, wait for testing and then leave the system after acceptance or rejection of the entire batches or lots. Hence the arrival of incoming lots can be considered as Poisson input and the service can be considered as exponential output. Many authors have contributed towards Queueing theory. But the literature is scarce in applying the Queueing theory into Quality control practice. Hence an attempt has been made to develop and design quality and efficiency domain for a production process with a Queueing system. In managing a production process with queuing system, the important and desirable is to determine the expected number of Quality control inspectors (C). The main objective of the paper is to find the number c such that it balances the quality and cost of inspection. In a steady state queuing system, $C = r + \Delta$, where $r = \frac{\lambda}{\mu}$, $\Delta$ = number of additional inspectors or servers needed. The constant c mainly depends on the offered load r. Hence one has to find the smallest c such that $1 - W_q \leq \alpha$. Many authors have contributed towards Queuing theory and Sampling plans. Donald Gross *et al*. [1] have published a collection of Queuing theory and models in their text Fundamentals of Queuing theory. Huei-Wen Ferng and Jin-Fu Chang [2] have published The departure process of discrete-time Queueing systems with Markovian type inputs. S.Murali and K.S.Ramaswami [3] have published an article A Numerical frame work for Solving D/G(A,N/A,Q)/ $1/Q_{max}$ of Discrete time Queues. Devaarul S and Santi T [4] have contributed towards Group Sampling Plans for a special type of production process. K.S. Ramaswami [5] has studied Bulk Queueing models with controllable services, state dependent arrivals and multiple vacations. Martin J. Beckmann [6] have published Making the customers wait: the optimal number of servers in a Queueing system. Dieter Fiems and Herwig Bruneel [7] have published Analysis of a Discrete-Time Queueing System with Timed Vacations. Hideaki

**Corresponding Author:** S. Murali, Department of Mathematics, Jansons Institute of Technology, Coimbatore-641 659, Tamil Nadu, India.

Takagi and Kin K. Leung [8] have published Analysis of a discrete-time Queueing system with time-limited service. Igor N. Kovalenko [9] have published Rare events in queueing systems. Kiyoshi Muto *et al*. [10] Lattice path counting and M/M/C Queueing systems.

## METHOD OF DESIGNING

**Algorithm for Sentencing the Lots in a M/M/C System:** Determine the parameters n, k and C by using Operating characteristic and Non-Delayed functions.

- Draw a random sample of size n from a lot or a batch of the system size k.
- Find the number of defective components in the sample. Let it be d.
- If $d \le c$ in each batch or group (k), accept the entire lots in the system.
- If $d > c$ reject the lots and screen the entire lots in the system.

For the above algorithm the operating characteristic function of group acceptance sampling plans is derived and is given in the following theorem.

**Theorem:** The Probability of acceptance of a group sampling plan for a M/M/C model is a function of np, c and $k = \frac{r}{1-r}$ and is given as

$$Pa(p) = \left\{\sum_{x=0}^{c} \frac{e^{-np}(np)^x}{x!}\right\}^k$$

**Proof:**

Let 'd' be the number of defectives found in each lots.
Let 'n' be the sample size of each lot.
Let 'c' be the acceptance constant. Then by using the given algorithm,

A single lot is accepted if $d \le c$. The entire system is accepted if $d \le c$ for k lots in the group.
$Pa(p) = \{P(d=0) \text{ or } P(d=1) \text{ or } .... .... P(d=c)\}$ ... ... ... up to k lots

$$= \{P(d \le c)\}^k$$

Therefore

$$Pa(p) = \left\{\sum_{x=0}^{c} \frac{e^{-np}(np)^x}{x!}\right\}^k$$

where, k = Expected number of arrivals in the system.

$k = \frac{r}{1-r}$ Where, $r = \frac{\lambda}{\mu}$, $\rho = \frac{r}{c} = \frac{\lambda}{c\mu}$

For a M/M/C model,

$$k = r + \left(\frac{r^c \rho}{c!\,(1-\rho)^2}\right) p_0$$

$$p_0 = \left[\sum_{n=0}^{c-1} \frac{r^n}{n!} + \sum_{n=c}^{\infty} \frac{r^n}{c^{n-c} c!}\right]^{-1}$$

Hence the proof.

**Advantages:** The main advantages of the above algorithm are the entire lot in a system is sentenced based on the expected number of arrivals and group sampling plans. And also the number of group is decided based upon the traffic intensity.

**Conditions for Application:**

- The production process should be stable and steady.
- The production process should be able to deliver products in batches or lots.
- The testing is done by the quality control inspectors.

**Measure of Sampling Plans for the Queuing System**

**Operating Characteristics Function:** The Operating Characteristics function is a measure of Probability of acceptance of the entire system for the known traffic intensity.

$$Pa(p) = \left\{\sum_{x=0}^{c} \frac{e^{-np}(np)^x}{x!}\right\}^k$$

where $k = \frac{r}{1-r}$

$$k = r + \left(\frac{r^c \rho}{c!\,(1-\rho)^2}\right) p_0$$

r = Traffic intensity of the system

**Average Outgoing Quality of Each Lot in the System:**

AOQ = p.Pa(p)

where p = fraction defectives of the system

**Average Outgoing Quality of Entire System:**

$$AOQ = [p.Pa(p)]^{\frac{r}{1-r}}$$

**Average Sample Number of Each Lot:**

ASN = n

**Average Sample Number of Entire Queuing System:**

$$ASN = n\,E(n). = n\left(\frac{r}{1-r}\right)$$

## RESULTS AND DISCUSSION

**Designing M/M/C Queueing system Sampling Plans:** Let $\alpha$ be the probability of non delayed lots inspection then one must have

$$1 - D(c,r) \geq 1 - \alpha \qquad (1)$$

$$\rho - \left[\frac{r^c \rho^0}{c!(1-\rho)}\right] \geq 0.95 \qquad (2)$$

Now our aim is to find minimum number of Quality control inspectors (c) for the known $\alpha$. For the known values of $\alpha$, tables are constructed for easy selection of number of inspectors. Similarly for the given values of Acceptable Quality Levels AQL, tables are constructed for easy selection of parameters of the sampling plans. Using equations (1) and (2), Tables (1) and (2) are constructed.

**Illustration:** A Quality Control system with M/M/C model consists of the traffic intensity r = 2.5 and the system should consist of at least 90% non delayed Ques. Determine the number of inspectors required for the testing purpose.

**Solution:**

It is given that r = 2.5 and p = 90%

Therefore from Table (2), c = 5.

Table 1: Number of Inspectors required for the M/M/C system for the known r and p.

| r | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
|---|---|---|---|---|---|---|
| p | c | c | c | c | c | c |
| 0.9 | 5 | 6 | 6 | 7 | 7 | 8 |
| 0.91 | 5 | 6 | 6 | 7 | 7 | 8 |
| 0.92 | 5 | 6 | 6 | 7 | 7 | 8 |
| 0.93 | 5 | 6 | 6 | 7 | 8 | 8 |
| 0.94 | 5 | 6 | 7 | 7 | 8 | 8 |
| 0.95 | 5 | 6 | 8 | 7 | 8 | 9 |
| 0.96 | 6 | 6 | 7 | 8 | 8 | 9 |
| 0.97 | 6 | 7 | 7 | 8 | 8 | 9 |
| 0.98 | 6 | 7 | 8 | 8 | 9 | 9 |
| 0.99 | 7 | 7 | 8 | 9 | 10 | 10 |

Table 2: Number of Quality control inspectors for the known r and p

| r | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
|---|---|---|---|---|---|---|
| p | c | c | c | c | c | c |
| 0.9 | 5 | 6 | 6 | 7 | 7 | 8 |
| 0.91 | 5 | 6 | 6 | 7 | 7 | 8 |
| 0.92 | 5 | 6 | 6 | 7 | 7 | 8 |
| 0.93 | 5 | 6 | 6 | 7 | 8 | 8 |
| 0.94 | 5 | 6 | 7 | 7 | 8 | 8 |
| 0.95 | 5 | 6 | 8 | 7 | 8 | 9 |
| 0.96 | 6 | 6 | 7 | 8 | 8 | 9 |
| 0.97 | 6 | 7 | 7 | 8 | 8 | 9 |
| 0.98 | 6 | 7 | 8 | 8 | 9 | 9 |
| 0.99 | 7 | 7 | 8 | 9 | 10 | 10 |

Hence the number of inspectors required is 5.

**Designing the Group Sampling Plans Indexed Through AQL for M/M/C Model:** The Probability of acceptance of the system is

$$Pa(p) = \left\{\sum_{x=0}^{c} \frac{e^{-np}(np)^x}{x!}\right\}^k$$

where $k = \frac{r}{1-r}$

r = Traffic intensity of the system

Now, one has to find the parameters n, c and K for the Known $P_a(p)$ and fraction defectives p such that $Pa(p) \geq 1 - i\alpha$.

A computer program is written to solve the above non-linear equation and hence the parametric values are found and tabulated in Tables (3) and (4).

**Illustration:** A Quality Control system with M/M/C model consists of the traffic intensity r = 0.5 and the system should consist of atleast 95% acceptance at AQL=1% when c=3. Determine the parameters of the Group Sampling Plans.

Table 3: Values of c and np for the known values of r

| | r=0.1 | r=0.15 | r=0.2 | r=0.25 | r=0.3 | r=0.35 | r=0.4 | r=0.45 | r=0.5 |
|---|---|---|---|---|---|---|---|---|---|
| c | np | np | np | np | np | np | np | np | np |
| 1 | 1 | | | | | | | | |
| 2 | 1.5 | 1 | 1 | 1 | 1 | 1 | | | |
| 3 | 2.5 | 2 | 1.5 | 1.5 | 1.5 | 1.5 | 1 | 1 | 1 |
| 4 | 3.5 | 3 | 2.5 | 2 | 2 | 2 | 2 | 1.5 | 1.5 |
| 5 | 4 | 3.5 | 3.5 | 3 | 3 | 2.5 | 2.5 | 2.5 | 2 |
| 6 | 5 | 4.5 | 4 | 4 | 3.5 | 3.5 | 3 | 3 | 3 |
| 7 | 6 | 5.5 | 5 | 4.5 | 4.5 | 4 | 4 | 3.5 | 3.5 |
| 8 | 7 | 6 | 5.5 | 5.5 | 5 | 5 | 4.5 | 4.5 | 4 |
| 9 | 8 | 7 | 6.5 | 6 | 6 | 5.5 | 5.5 | 5 | 5 |
| 10 | 9 | 8 | 7.5 | 7 | 6.5 | 6.5 | 6 | 6 | 5.5 |
| 11 | 10 | 9 | 8.5 | 8 | 7.5 | 7 | 7 | 6.5 | 6.5 |
| 12 | | 10 | 9 | 8.5 | 8.5 | 8 | 7.5 | 7.5 | 7 |
| 13 | | | 10 | 9.5 | 9 | 8.5 | 8.5 | 8 | 8 |
| 14 | | | | | 10 | 9.5 | 9.5 | 9 | 8.5 |
| 15 | | | | | | | 10 | 10 | 9.5 |

Table 4: Values of c, np for known values of r

| | r=0.55 | r=0.6 | r=0.65 | r=0.7 | r=0.75 | r=0.8 | r=0.85 | r=0.9 | r=0.95 |
|---|---|---|---|---|---|---|---|---|---|
| c | np | np | np | np | np | np | np | np | np |
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | 1 | 1 | 1 | 1 | | | | | |
| 4 | 1.5 | 1.5 | 1.5 | 1.5 | 1 | 1 | 1 | 1 | |
| 5 | 2 | 2 | 2 | 2 | 1.5 | 1.5 | 1.5 | 1.5 | 5 |
| 6 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 2 | 2 | 6 |
| 7 | 3.5 | 3.5 | 3 | 3 | 3 | 2.5 | 2.5 | 2.5 | 7 |
| 8 | 4 | 4 | 4 | 3.5 | 3.5 | 3.5 | 3 | 3 | 8 |
| 9 | 5 | 4.5 | 4.5 | 4.5 | 4 | 4 | 3.5 | 3.5 | 9 |
| 10 | 5.5 | 5.5 | 5 | 5 | 4.5 | 4.5 | 4.5 | 4 | 10 |
| 11 | 6 | 6 | 6 | 5.5 | 5.5 | 5 | 5 | 4.5 | 11 |
| 12 | 7 | 6.5 | 6.5 | 6.5 | 6 | 6 | 5.5 | 5.5 | 12 |
| 13 | 7.5 | 7.5 | 7.5 | 7 | 7 | 6.5 | 6.5 | 6 | 13 |
| 14 | 8.5 | 8 | 8 | 8 | 7.5 | 7 | 7 | 6.5 | 14 |
| 15 | 9 | 9 | 8.5 | 8.5 | 8 | 8 | 7.5 | 7 | 15 |
| 16 | 10 | 10 | 9.5 | 9 | 9 | 8.5 | 8.5 | 8 | 16 |
| 17 | | | | 10 | 9.5 | 9.5 | 9 | 8.5 | 17 |
| 18 | | | | | | 10 | 10 | 9.5 | 18 |
| 19 | | | | | | | | 10 | 19 |
| 20 | | | | | | | | | 20 |

**Solution:** It is given that r = 0.5 and $1 - \alpha = 0.95$, therefore by using table (3) we get np = 1.

It is given that p = 1%, therefore when np = 1 which implies that n = 100.

## CONCLUSION

In this article Group sampling plans are developed for an M/M/C production process. A novel algorithm is given for sentencing the group of batches or lots which comprises of components. The sampling plans are developed based on the expected number of arrivals in the system. The measures of sampling plans are given for M/M/C type of production process. Tables are constructed to select the number of quality control inspectors (servers) and to select the parameters of the sampling plans.

## REFERENCES


1. Donald Gross, John F. Shortle, James M. Thompson and Carl M. Harris, 2008. Fundamentals of Queueing Theory, John Wiley & sons, IV edition, New Delhi.
2. Huei-Wen Ferng and Jin-Fu Chang, 2000. The departure process of discrete-time queueing systems with Markovian type inputs, Queueing Systems, 36 :201-220.

3. Murali, S. and K.S. Ramaswami, 2014. A Numerical frame work for Solving D/G(A,N/A,Q)/ 1/$Q_{max}$ of Discrete time Queues, International Journal of Applied Engineering Research, 9(22): 11961-11968.
4. Devaarul, S. and T. Santi, 2014. A Study on Group Sampling plans indexed through Inflection point, National Conference on Recent advances in Statistical models and methods, Department of Statistics, Government Arts College and Coimbatore, India.
5. Ramaswami, K.S., 2005. Bulk Queuing Models with Controllable Services, State Dependent Arrivals and multiple vacations, Ph.D. Thesis, Department of Mathematics and Computer Applications, P.S.G. College of Technology, Coimbatore - 641 004, India.
6. Martin J. Beckmann, 1994. Making the customers wait:the optimal number of servers in a queueing system, OR Spektrum, 16: 77-79.
7. Dieter Fiems and Herwig Brunel, Analysis of a Discrete-Time Queueing System with Timed Vacations, Queuing Systems, 42: 243-254.
8. Hideaki Takagi and Kin K. Leung, 1994. Analysis of a discrete-time queueing system with time-limited service, Queueing Systems, 18: 183-197.
9. Igor N. Kovalenko, 1994. Rare events in Queueing systems - A survey, Queueing Systems, 16: 1-49.
10. Kiyoshi Muto, Haruo Miyazaki, Yoichi Sek, Yoshihiro Kimura and Yukio Shibata, 1995. Lattice path counting and M/M/C Queueing systems, Queuing Systems, 19: 193-214.