Investigating the Impact of Inflation Frequency Path on the Structure of Labor Market Demand in Iran

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Abstract: The labor market is one of the four economic markets which possesses a key role in adjusting the relationship between workforce demand and supply as well as the balance in macroeconomic variables such as employment. Hence, the basic question which is the main focus of the present paper is whether the current frequencies in inflation impact labor market demand or not? In order to answer this question based on the previous literature and theoretical concepts, labor market demand function, wage level function, gross domestic product (GDP) and the working population are considered. In this way, first labor market demand frequencies are identified for the period between 1996 and 2012 and then in order to determine the effects of inflation frequencies on labor market demand, this variable is entered into the function and it will be calculated again. The results show that wage level, gross domestic product, working population and inflation have a positive effect on the labor market demand and inflation frequencies influence labor market demand. Hence, if inflation were entered into the demand function, labor market demand structure functions would be increased and management of this market would be harder. Hence, in the case that inflation frequencies are managed it might be possible to manage the present frequencies in labor market demand.

Key words: JEL Classification (E31;C32;C61) • Inflation • Labor Market Demand • Frequency • Fourier series

INTRODUCTION

Each market is characterized through supply and demand mechanisms. The labor market, among the four economic markets, possesses a key role in adjusting the relationship between workforce demand and supply as well as the balance in macroeconomic variables such as employment. Regarding workforce demand, workforce productivity, the technology used, as well as relative labor prices and capital are considered. The consequences and effects of the socioeconomic balance in the labor market attracted the attention of economists as well as policy makers. One of the direct consequences of such a balance is preventing costs and expenses posed by unemployment. Unfortunately, in Iran workforce demand has not grown along with the workforce supply since structural barriers has led to reduced economic growth in recent decades and even in some cases this growth has been minuscule. On the other hand, because of the imported technology (which are often costly) as well as several other organizational, political reasons and the manipulation on the part of the government, the real cost of work has been higher and the capital cost has been lower than their real value since state funds in the form of subsidies along with troublesome rules as well as a young population and unproductive work in developing countries have led to the fact that the workforce supply is higher than its demand [1].

A type of business cycle modeling in economy started with the works of Samuelsson (1939), Hicks (1950), Godwin (1949) and others, in a way that these models are considered in two modes; namely continuous and discrete times. If the time is considered continuous, these models will yield differential equations; on the other hand, if time is considered discrete, the outcome of these systems will be subtractive equations. Solving these equations some production outputs will be provided in a way that they contain sine or cosine terms with a certain frequency, which in turn confirms the presence of business cycles in
economy. It is worth mentioning that in these systems the presence of a business cycle is truly and literally proved; that is terms that exactly contain frequency and amplitude. In other words, these terms will surface:

\[ y(t) = \sum c_m \cos(mt) + \sum d_m \sin(mt) \]

The foundation of these cycles is the multiplier-acceleration model.

In this paper the main objective is to evaluate the frequency impact of inflation on labor market demand in Iran. In order to realize this objective, this paper is organized as follows: first a brief introduction will be presented; in section II the previous literature is offered. Section III the proposed model is explicitly presented. In section IV the model estimations are done and finally in section V the conclusions will be stated [2].

**Review of the Literature:** Many studies have been conducted concerning the discussion of Labor market demand. Therefore, attempt has been made to mention the studies in the literature.

**Empirical Studies:** Pesaran and Smith (1995) The theoretical results on the properties of the four procedures are illustrated by UK labour demand functions for 38 industries over 30 years. Jorgenson et al. (2008) model U.S. labor supply and demand over the next 25 years. Despite the anticipated aging of the population, moderate population growth will provide growing supplies of labor well into the 21st century. Palazuelos and Fernández (2009) presents an explanation of the causes of the slowdown in growth in labour productivity in European economies in recent decades. In first instance, the weakness of domestic demand is what determines this slowdown in productivity. However, differences with the (mediocre) rates of growth of productivity between European countries are also related to the specific features of their respective labour markets because, in a context of weak domestic demand, there is a trade-off between employment and productivity. Kurose(2013) The purpose is to provide a coherent framework to explain the unusual phenomena of employment, real wage and profit share observed in industrialised economies since the 1980s, in relation to the speed of demand saturation. show that faster demand saturation tends to accelerate the growth of employment but decelerate the growth of the real wage. Furthermore, show that faster demand saturation tends to increase the profit share and the share eventually converges irrespective of the difference in the speed of demand saturation. Mouelhi and Ghazali(2012) This paper divides labor into skilled and unskilled categories in order to analyze the effects of trade policies on labor demand elasticities by skill in Tunisia. We use dynamic panel techniques to estimate a model of employment determination, which incorporates the effects of trade and takes into account the delay of labor adjustment. Results suggest that a decrease in trade protection in Tunisia increases the elasticity of unskilled labor demand while it contributes to the decrease in the elasticity of skilled labor demand. Ghazali (2009) emphasizes the existence of a positive and statistically significant relationship between trade openness and wage inequality between skilled and unskilled workers. Greater labor demand elasticities might be an indirect channel through which trade effects on wage differentials transit [3].

**The Model Explanation**

**Frequency Analysis**

**Inflation and Unemployment: the Standard Treatment:** The standard treatment of the relationship between inflation and unemployment has well been studied by mathematical economists such as Chiang, Pemberton and Rau and Todorova. The original Phillips relation shows that the rate of inflation is negatively related to the level of unemployment and positively to the expected rate of inflation such that

\[ p = \alpha - \beta U + h \pi \quad \alpha, \beta \geq 0, \quad 0(h \leq 1) \]  \hspace{1cm} (1)

where \( p \) is the rate of growth of the price level, i.e., the inflation rate, \( U \) is the rate of unemployment and \( \pi \) denotes the expected rate of inflation. Thus the expectation of higher inflation shapes the behavior of firms and individuals in a way that stimulates inflation, indeed (expecting prices to rise, they might decide to buy more presently). As people expect inflation to go down (as a result of appropriate government policies, for example), this, indeed, brings actual inflation down. This version of the Phillips relation that accounts for the expected rate of inflation is called the expectations-augmented Phillips relation. The adaptive expectations hypothesis further shows how inflationary expectations are formed. The equation

\[ \frac{d\pi}{dt} = j(\hat{p} - \pi) \quad 0 \leq j \leq 1 \]  \hspace{1cm} (2)

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illustrates that when the actual rate of inflation exceeds the expected one, this nurtures people’s expectations so 
\( \frac{d\pi}{dt} \). In the opposite case, if the actual inflation is below the expected one, this makes people believe that inflation would go down so is reduced. If the projected and the real inflation turn out to be equal, people do not expect a change in the level of inflation. \( \pi \). There is also the reverse effect, that of inflation on unemployment. When inflation is high for too long, this may discourage people from saving, consequently reduce aggregate investment and increase the rate of unemployment. We can write

\[
\frac{dU}{dt} = -k(\bar{m} - \bar{P})
\]

or unemployment increases proportionally with real money where \( \bar{m} \) is the rate of growth of nominal money. The expression \((\bar{m} - \bar{P})\) gives the rate of growth of real money, or the difference between the growth rate of nominal money and the rate of inflation

\[
m - \bar{P} = \frac{m'}{m} - \frac{\bar{P}'}{\bar{P}} = \frac{m - \bar{P}}{\bar{P}}
\]

where real money is nominal money divided by the average price level in the economy. The model then becomes

\[
\Delta = \alpha - \beta U + h \pi \quad \alpha, \beta > 0 , \quad 0 < h < 1
\]

(expectations-augmented Philips relation)

\[
\frac{d\pi}{dt} = j(\bar{P} - \bar{P}) \quad 0 \leq j \leq 1
\]

(adaptive expectations)

\[
\frac{dU}{dt} = -k(\bar{m} - \bar{P}) \quad k > 0
\]

(monetary policy)

We solve this model by substituting the first equation into the second which gives

\[
\frac{d\pi}{dt} = j(\alpha - \beta U) + j(h - 1)\pi
\]

Differentiating further with respect to time \( t \),

\[
\frac{d^2\pi}{dt^2} = -j\beta \frac{dU}{dt} + j(h - 1)\frac{d\pi}{dt}
\]

and substituting for \( \frac{dU}{dt} \) we obtain

\[
\frac{d^2\pi}{dt^2} = j\beta k(\bar{m} - \bar{P}) + j(h - 1)\frac{d\pi}{dt}
\]

where the second equation of the model implies

\[
\dot{\pi} = j\beta k \cdot \frac{d\pi}{dt}
\]

Substituting this last expression for \( \dot{\pi} \) we obtain

\[
\frac{d^2\pi}{dt^2} = j\beta k \left[ \frac{m - \bar{P}}{j \frac{d\pi}{dt}} + \pi \right] + j(h - 1)\frac{d\pi}{dt}
\]

This is a second-order differential equation in which transforms into

\[
\frac{d^2\pi}{dt^2} = j\beta k \left[ \frac{m - \bar{P}}{j \frac{d\pi}{dt}} + \pi \right] + j(h - 1)\frac{d\pi}{dt}
\]

or alternatively

\[
\pi'' + \left[ \beta k + j(1 - h) \right] \pi' + j\beta k \pi = j\beta k \bar{m}
\]

Given the properties of second-order differential equations, we have the following parameters

\[
a_1 = \beta k + j(1 - h) \quad a_2 = j\beta k \quad b = j\beta k \bar{m}
\]

The coefficients 1 and 2 are both positive in view of the signs of the parameters. We find the equilibrium rate of expected inflation to be the particular integral

\[
\bar{\pi} = \frac{b}{a_2} = \bar{m}
\]

Hence, the intertemporal equilibrium of the expected rate of inflation is exactly the rate of growth of nominal money. In order to establish the time path of we need to find the characteristic roots of the differential equation which we can do using the formula

\[
\rho_{1,2} = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2}
\]

The time path of \( \pi \) would depend on the particular values of the parameters. Once we find this time path we might be able to determine that of unemployment \( U \) or the rate of inflation \( \dot{\pi} \) [4].
Inflation and Unemployment: an Extended Model: In his book Macroeconomics Blanchard offers an alternative treatment of the relationship between inflation and unemployment. He introduces in the model the natural rate of unemployment \( U_n \), at which the actual and the expected inflation rates are equal. The rate of change of the inflation rate \( \dot{P} \) is proportional to the difference between the actual unemployment rate \( U \) and the natural rate of unemployment \( U_n \), such that

\[
\frac{dP}{dt} = -\alpha(U - U_n), \quad \alpha > 0 \tag{17}
\]

Therefore, when \( U > U_n \), that is, the actual rate of unemployment exceeds the natural rate, the inflation rate decreases and when \( U < U_n \) the inflation rate increases.

The intuitive logic behind this is that in bad economic times when many people are laid off, prices tend to fall. At this point the actual unemployment would exceed the normal levels. In times of a boom in the business cycle the rate of actual unemployment would be rather low but high aggregate demand would push prices up. Blanchard’s equation reveals an important relation as it gives another way of thinking about the Phillips curve in terms of the actual and the natural unemployment rates and the change in the inflation rate. Furthermore, it introduces the natural rate of unemployment as it relates to the non-accelerating-inflation rate of unemployment (or NAIRU) the rate of unemployment required to keep the inflation rate constant. We solve this alternative model of the relationship between inflation and unemployment by assuming that \( U_n \) is constant and that at any given time the actual unemployment rate \( U \) is determined by aggregate demand which, on its own, depends on the real value of money supply given by nominal money supply \( M \) divided by the average price level \( P \). Thus unemployment is negatively related to real money supply \( \frac{M}{P} \) according to the relationship

\[
U = \gamma - \beta \ln \left( \frac{M}{P} \right), \quad \beta, \gamma > 0 \tag{18}
\]

We solve by differentiating the first equation

\[
\frac{d^2P}{dt^2} = -\alpha \frac{dU}{dt} \tag{19}
\]

and the second equation to obtain \( \frac{dU}{dt} \)

\[
\frac{dU}{dt} = -\beta \frac{d}{dt} \ln \left( \frac{M}{P} \right) = -\beta \left[ \frac{d}{dt} \ln M - \frac{d}{dt} \ln P \right] = -\beta \left( \frac{M}{P} \dot{P} \right) \tag{20}
\]

We assume that the growth rate of nominal money supply \( \dot{m} \) is constant which could be in accordance with systematic government planning or monetary policy. The equation that obtains is identical to the monetary-policy equation introduced in the standard treatment of the Phillips curve. Combining the two results yields

\[
\frac{d^2P}{dt^2} = -\alpha \frac{dU}{dt} = \alpha \beta \left( \frac{M}{P} \dot{P} \right)
\]

\[
\frac{d^2P}{dt^2} + \alpha \beta \dot{P} = \alpha \beta M \tag{21}
\]

which is a second-order differential equation in inflation rate \( \dot{P} \). Solving the differential equation, we have \( a_1 = 0 \), \( a_2 = \alpha \beta \), \( b = \alpha \beta M \), Hence, the particular integral is \( \hat{\dot{P}} = m \) and the characteristic equation is

\[
r^2 + \alpha \beta = 0 \tag{22}
\]

\[
r_{1,2} = \pm \sqrt{\alpha \beta} i \tag{23}
\]

where \( h = 0 \) and \( \nu = \sqrt{\alpha \beta} \).

Thus the general solution involves complex roots and takes the form

\[
P(t) = \hat{m} + e^{0} \left( B_1 \cos \sqrt{\alpha \beta} t + B_2 \sin \sqrt{\alpha \beta} t \right)
\]

\[
= \hat{m} + B_1 \cos t \sqrt{\alpha \beta} + B_2 \sin t \sqrt{\alpha \beta} \tag{24}
\]

Similar to the standard model we can study the dynamic stability of actual inflation. Since, the function of inflation rate displays uniform fluctuations around the rate of growth of money supply which gives the equilibrium level of inflation. Since the growth rate of nominal money supply depends on government policies and changes with those, it is a moving equilibrium. Such fluctuating time path around the intertemporal equilibrium can be graphed as in Figure 1. Although the time path is not convergent, monetary policy can somewhat steer inflation and limit it within a tunnel as it fluctuates around \( \hat{m} \). Given the premises of the model and the values of the parameters, a divergent time path and, therefore, an uncontrollable level of inflation are impossible [5].
To find the time path of unemployment $U$ as the next step we express $\frac{dP}{dt}$ as

$$\frac{dP}{dt} = \sqrt{\alpha \beta} \left( -B_1 \sin t \sqrt{\alpha \beta} + B_2 \cos t \sqrt{\alpha \beta} \right)$$  \hspace{1cm} (25)$$

and substitute it into

$$U = U_n - \frac{1}{\alpha} \frac{dP}{dt}$$

$$= U_n - \sqrt{\frac{\beta}{\alpha}} \left( -B_1 \sin t \sqrt{\alpha \beta} + B_2 \cos t \sqrt{\alpha \beta} \right)$$

$$= U_n + B_1 \frac{\beta}{\alpha} \sin t \sqrt{\alpha \beta} - B_2 \frac{\beta}{\alpha} \cos t \sqrt{\alpha \beta}$$  \hspace{1cm} (26)$$

where the constants $B_1$ and $B_2$ have not been definitized. It follows that, similar to the inflation rate, the unemployment rate displays regular fluctuations but its intertemporal equilibrium is the natural rate of unemployment. Since this is the rate at which expected and actual inflation are equal, we can view intertemporal equilibrium as the state in which expectations coincide with reality. Since again we have $h = 0$, the time path of unemployment is neither convergent, nor divergent. It follows, therefore, that with the passage of time actual unemployment cannot substantially deviate from the natural rate of unemployment [6].

**Fourier Series Analysis:** The advent of Fourier analysis goes back to a long time ago, different personalities and physical examinations were involved. The idea of using trigonometric sums, namely the sine and cosine sums or related alternating harmonic complex exponentials for describing periodic phenomena goes back to the Babylonian times. They used these structures for predicting astronomical occurrences. The starting date for this topic was in 1748 with the works of Euler. He investigated the movements of vibrating strings. For each given time normal faces of sine functions are compatible. Euler showed that if the arrangement of the vibrating string in a point in time were the linear combination of these natural faces, its arrangement in all the following points in time would be the same. Moreover, Euler showed that the coefficients of the linear combination in each point in time could be obtained from the coefficients of the linear combinations of the previous points in time [7].

Jean Baptiste Joseph Fourier (21 March 1768 – 16 May 1830) published his thoughts half a century later. He completed his work in 1807. He found out that harmonic related sine series are useful in depicting temperature distribution in a material. He also claimed that any periodic signal could be depicted using these series. Although Fourier’s work in this field was noteworthy, a lot of its supportive information was discovered by others. Moreover, Fourier’s mathematical calculations were not exact. The conditions in which a periodic signal could be depicted using Fourier series were stated by Dirichlet in 1829.

Hence, Fourier really did not do a lot of work regarding the mathematical theory of Fourier series. However, he had this clear vision to see the potential ability to use these series for presentation. He depicted non-periodic signals not as a product of weighted sums of harmonic sines rather as weighted integrals of non-harmonic sines. Fourier’s integral or transformation is one of the strongest equation analysis tools similar to Fourier series [8].

**Depicting Fourier Series of Periodic Signals in Time $T$ with Period $T$:** A signal is periodic in which for each positive non-zero $T$ we have:

$$f(t) = f(t + T)$$ \hspace{1cm} (23)$$

The base period of $f(t)$ is the smallest non-zero value of $T$ which satisfies the above relation and $\frac{\omega_0 T}{2\pi}$ is called the basic frequency. Hence, the Fourier series of the function $f(t)$ is shown as:

$$f(t) = \frac{a_0}{2} + \sum_m a_m \cos(m \omega_0 t) + \sum_m b_m \sin(m \omega_0 t)$$

$$a_0 = \frac{\omega_0}{\pi} \int_0^{2\pi} f(t) dt$$

$$a_m = \frac{\omega_0}{\pi} \int_0^{2\pi} f(t) \cos(m \omega_0 t) dt$$

$$b_m = \frac{\omega_0}{\pi} \int_0^{2\pi} f(t) \sin(m \omega_0 t) dt$$
Model Estimation

Model Estimation Regardless of Inflation: By reviewing the previous researches and studies on workforce demand, different effective factors on labor market demand can be identified. Hence in the current study the number of employees (LN) is considered as a function of the number of employees in the previous period (LN_{t-1}), wage level (Lw) and gross domestic product (LY). These variables are logarithmic and cover the time period between 1996 and 2012. Hence labor market demand function is presented as follows.

\[ LN = F(LN_{t-1}, LW, LY) \]

Generally the logarithmic function is used for estimating the demand function. The advantage of this function form over other forms is that it shows the elasticity coefficients and it has applied interpretation.

\[ LN_t = \alpha + \beta_1 LN_{t-1} + \beta_2 LW + \beta_3 LY \]

Using the genetic algorithm the function (25) is estimated and the results are shown below.

Model Estimation Taking into Account Inflation: Due to the explicit model, which proves the inflation path frequency, its impact on the course of labor market demand is investigated. For this purpose, a variable inflation and re-enter the labor market demand function is estimated.

\[ LN_t = \alpha + \beta_1 LN_{t-1} + \beta_2 LW + \beta_3 LY + \beta_4 LP \]

LP is the inflation rate. Using the genetic algorithm the function (26) is estimated and the results are shown below.
\[ LN_t = 2.77LN_{t-1} + 3.52\,LY + 3.75LW + 2.11LP - 1.089 \]

Using MATLAB software application and Curve Fitting method the results for labor market demand function path for the period between 1996 and 2012 are as follows:

\[
f(x) = a_0 + a_1 \cos(x\omega) + b_1 \sin(x\omega) + a_2 \cos(2x\omega) + b_2 \sin(2x\omega) + a_3 \cos(3x\omega) + b_3 \sin(3x\omega) \\
+ a_4 \cos(4x\omega) + b_4 \sin(4x\omega) + a_5 \cos(5x\omega) + b_5 \sin(5x\omega) + a_6 \cos(6x\omega) + b_6 \sin(6x\omega) \\
+ a_7 \cos(7x\omega) + b_7 \sin(7x\omega) + a_8 \cos(8x\omega) + b_8 \sin(8x\omega)
\]

**CONCLUSION**

In any economy, in the long run, the demand for labor is a very important factor in macroeconomic policy. The labor market is one of the four economic markets which possesses a key role in adjusting the relationship between workforce demand and supply as well as the balance in macroeconomic variables such as employment. Hence, the basic question which is the main focus of the present paper is whether the current frequencies in inflation impact labor market demand or not? In order to answer this question based on the previous literature and theoretical concepts, labor market demand function, wage level function, gross domestic product (GDP) and the working population are considered. In this way, first labor market demand frequencies are identified for the period between 1996 and 2012 and then in order to determine the effects of inflation frequencies on labor market demand, this variable is entered into the function and it will be calculated again. The results show that wage level, gross domestic product, working population and inflation have a positive effect on the labor market demand and inflation frequencies influence labor market demand. Hence, if inflation were entered into the demand function, labor market demand structure functions would be increased and management of this market would be harder. Hence, in the case that inflation frequencies are managed it might be possible to manage the present frequencies in labor market demand.

**REFERENCES**