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The Calculation of the Coefficient of Friction for Lining under Variable Load Levels

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Abstract: When grinding cement, ferrous and non-ferrous metals is commonly used drum ball mills. For example, the share of cement in grinding mill consumes more than 60% of the energy used to manufacture it. The material in the drum ball mill is crushed under the influence of grinding media (balls) moved in the cross-section drum mill due to shock and abrasive loads. It is known that if the material particles are larger than 1 mm, it is advisable to grind by punch and less than 1 mm by abrasion. Most often drum mill is divided by a perforated partition into two chambers-in the first grinding bodies are running in crushing hammer mode, while the second chamber abrades. Since at the end of the first chamber in a drum mill there is more than 50% of particles smaller than 1 mm it is necessary to create conditions of abrasive grinding bodies operation. As the mill drum is rotated with a certain constant frequency mode to regulate the movement of grinding bodies by other means, namely the size of the grinding bodies, the lining profile, load factor of each mill chamber. The most effective way to achieve the desired mode of motion of grinding media, is the change in the coefficient of friction between the grinding media and lining. This is achieved by a certain profile of lining. By changing the profile of the liner at each site of mills drum we provide conditions for the selectivity of the grinding process without changing the value of the load factor of the chamber grinding media, grinding media size and the frequency of rotation of the mill drum.

Key words: Ball drum mill . grinding media . grinding media motion mode . the load factor of mill chambers . lining . the transverse profile of the lining . coefficient of coupling . load circuit . the tangential . normal component of sector forces . slip loading . longitudinal motion of the grinding media . the speed of rotation

INTRODUCTION

In recent years, one of the main directions of improving the design of drum mills is an increase in their overall dimensions. The undoubted benefits of large-size mills are more hours of productivity, reduction of specific energy consumption, the availability of their nodes at their maintenance and repair, better terms of automation and control of milling process [1-3].

The main drawbacks that hinder the increase of dimensions of ball drum mills are as follows: complicating of mills nodes production, its transportation and installation, reducing reliability and complicating the lining fastening; worsening conditions for longitudinal classification of grinding media, increasing operating costs, increasing accuracy requirements for units and parts.

Despite the shortcomings in the nearest years a drum ball mill will remain the main unit when grinding different materials with productivity up to 800 t / h.

Further improvement of technological and operational performance of the ball drum mills will be based on the improvement of the design of lining and in-drum devices that would ensure the selectivity of the grinding process [4].

One solution is to create ball drum mills with a cross-longitudinal motion of grinding media [5-7]. Due to the fore-aft movement of the grinding bodies dead zones are broken down, intensifying mutual displacement of the grinding bodies and, as a consequence, the work of grinding is increased.

To create the desired mode of motion of grinding bodies in both transverse and longitudinal cross-section of mill drum it is necessary to calculate the coefficient of adhesion of grinding bodies taking into account the cross-section of the lining [8-10].

Knowing the value of the coefficient of friction we can specify any (desired) mode of grinding bodies in each section of the mill drum, thus creating the most effective conditions for the grinding process of the material.

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Conditions of the transversely-longitudinal motion of the grinding bodies are created in the mill drum by installing inclined interchamber partitions or by specially shaped liner [5-7].

Method of calculation of the coefficient of friction for ball drum mills with transverse motion of grinding bodies is presented in [8-10].

The main part: The calculation of the coefficients of friction and adhesion in the theory of tumbling mills has so far received undeservedly little attention.

Kryukov DK [10] proposed a graphical-analytical method for calculating the coefficient of friction at which slippage of grinding media is prevented. It should be recognized that the graphic-analytical method is currently the most useful for determining the effect of the profile of the lining on the friction coefficient.

Applying the known methods for calculating the coefficient of friction for mills with a longitudinal cross-traffic load is not possible, because none of them accounts for the longitudinal motion factor of grinding bodies and changes of the load level during the cycle.

Unlike graphoanalytical method, the basis of our analytical calculation method is analytical calculation of pressure force of grinding media and its components on the lining. Eventually, the friction coefficient is determined from the condition under which slippage of the lining against loading is prevented-it is ensured if the normal component of the pressure force will be more than tangential load, $k_c = F^{[tau]}/F^n$. It is known that the relationship between friction coefficient and adhesion friction must satisfy $k_c \le k_T$ [9, 10].

Thus, the coefficient of adhesion is the minimum coefficient of friction, wherein there is no slippage of the outer load layer against the liner.

We can say beforehand that $k_c \rightarrow k_T$ operation of grinding media and liner profile is the most rational. In general, the adhesion coefficient can be within $0<k_c>>1$.

The range of the coefficient of adhesion $0<k_c<k_T$ is provided by liners, where height and space of the projections are those that there is no load sliding. If $k_c>>k_T$ then the wrong lining profile is selected-load slips against the lining.

Cross-circuit of load: Thus, we consider a cross-circuit taken by load in an infinitely small time interval dt.

Circuit (Fig. 1), in accordance with the theory of Davis is limited to four distinctive areas: A_0B_0; A_0A_1; A_1B_1; B_1B_0. Line A_0B_0 is a circular arc with a radius equal to the radius of the mill drum R. Line A_1B_1 is also a circular arc with a radius $R_1 = kR$.

The dimensionless parameter K is calculated according to the [phi] and [Phi] (the equation Ossietzky-Kantorovich):

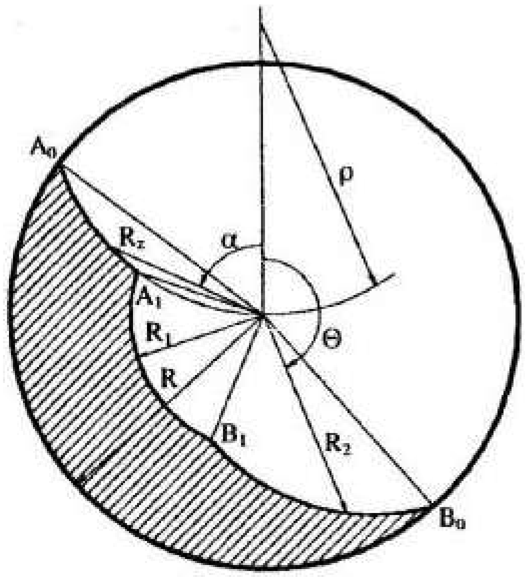

Fig. 1: The transverse load circuit

$$\pi\varphi\cos^2\alpha_0 = \left[\left(\frac{\pi}{2}-\alpha\right)\cos 2\alpha - \frac{\alpha}{2} + \frac{\sin 2\alpha}{2} + \frac{\sin 4\alpha}{8}\right]_{\alpha=\alpha_1}^{\alpha=\alpha_0} \quad (1)$$

here

$$\cos[alpha]_0 = [Phi^2],\ k = \cos[alpha]/\cos[alpha]_0$$

Line A_0 A_1 is a circular arc whose center is at the point M on the line with the vertical axis of the drum at a distance from the center equal to the radius [rho]

$$\rho = R/2\varphi^2 \quad (2)$$

Line B_0 B_1 is part of the spiral described by the equation

$$R_2 = 2\rho\cos\theta/3 \quad (3)$$

The division of load circuit into sectors. Arc A_0 B_0 of the load circuit is divided by the number of sectors. The more sectors, the more accurate are calculations. The spacing of sectors is equal to the space of plates forming lining-l_ϕ. Offset of lining l_ϕ has greater effect on the value k_c, the greater is the lining spacing l_ϕ. Total number of sectors d_c and the residue near the point B_0 l''_ϕ, $l''_\phi < l_\phi$, can be determined by the formulas:

$$d_c = \left[(2\pi - 4\alpha_0)R - l'_\phi\right]/l_\phi;\ l''_\phi = (2\pi - 4\alpha_0)R - l'_\phi - d_c l_\phi \quad (4)$$

here the square brackets denote the integer part of a number.

For each sector (Fig. 2), we need to calculate: $[alpha]_k$-the angle between the bisector of the sector and

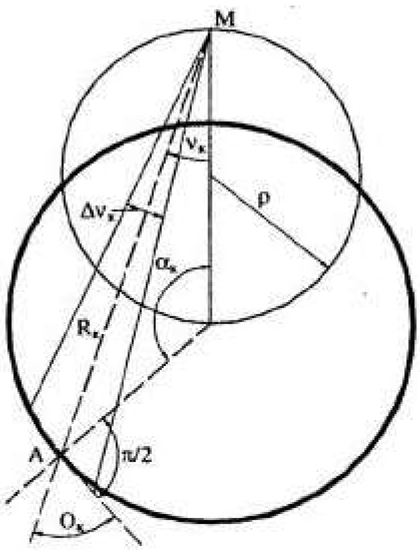

Fig. 2: Design model of the load cross-circuit

the vertical; v_k-sector angle; O_k-and the bisector of the angle between the tangent to the drum; index k-number of sectors.

Angle [alpha]$_k$ (Fig. 2) we calculate as:

$$\alpha_k = \alpha_0 + \begin{cases} l'_\phi / 2R, k = 1, \\ \left[(k - 1{,}5) l_\phi + l'_\phi \right] / R, 1<k<a+2, \\ \left(a l_\phi + l'_\phi + l''_\phi / 2 \right) / R, k = a + 2. \end{cases} \quad (5)$$

Radius R_k and sector angel v_k:

$$R_k = R\phi^{-2} \left[1 - 2\phi^2 \cos\alpha_k + \phi^4 \right]^{0,5} \quad (6)$$

$$\nu_k = \arcsin R \sin\alpha_k / R \quad (7)$$

If [alpha]$_k$>[p$_i$], then, in accordance with the calculation figure (Fig. 2), v_k<0.

Angle O_k is equal to:

$$O_k = \alpha_k + \nu_k - \pi / 2 \quad (8)$$

If each sector lining arc lies to the right of the vertical diameter then O_k>[p$_i$]/2 (Fig. 2).

To calculate Δv_k it is more convenient to use the formula:

$$\Delta\nu \approx l_k \sin O_k / R_k \quad (9)$$

Determination of sector strength: The strength of the layer pressure on grinding bodies in each sector is calculated as the centrifugal force of the mass of

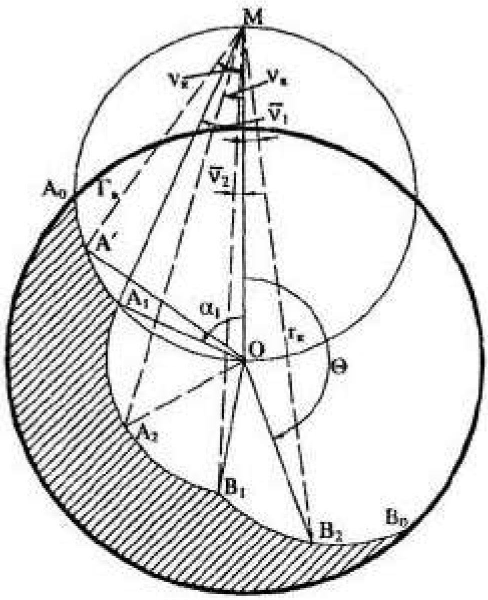

Fig. 3: Calculation of load circuit sectors

milling bodies attached to the plane of the lining with the center of rotation at the point M (Fig. 2)

$$F = m_k \omega^2 \rho = R m_k \omega^2 / 2\phi^2 \quad (10)$$

Mass of loading m_k, located on an arc sector with a central angle Δv_k, is equal:

$$m_k = \gamma \Delta\nu_k (R_k^3 - r_k^3) \quad (11)$$

The force generated by the mass m_k and effecting the unit of an arc lining sector length with a central angle Δv_k is equal to:

$$F_k = a_1 \Delta\nu_k (R_k^3 - r_k^3) \quad (12)$$

The internal circuit of the load is limited to three different sites: A_0A_1, A_1 B_1, B_1B_0. Obviously, the formula for calculation is also different.

The boundaries of each of the typical sectors of the load cross-circuit is defined by the angles $\overline{\nu}_1$ and $\overline{\nu}_2$, which are respectively equal to:

$$\overline{\nu}_1 = 0{,}5\pi - \alpha_1 \quad (13)$$

$$\overline{\nu}_2 = \arcsin \left[\frac{-\sin 3\alpha_1 \cos\alpha_1 /}{\left(1 + \cos^2\alpha_1 - 2\cos\alpha_1 \cos 3\alpha_1 \right)^{0,5}} \right] 0{,}5\pi - \alpha_1 \quad (14)$$

when calculating the length of the internal bisectors r_k there can be three options.

The first option, $v_K < \overline{v}_1$. In this case (Fig. 3) from OA'M we have:

$$r_k = R\cos v_k / \phi^2 \qquad (15)$$

The second option, $\overline{v}_1 < v_K < \overline{v}_2$. This is possible if the angle v is in the range A_2MO. From the triangle A_2OM, after appropriate transformations, we obtain:

$$r_k = R\begin{bmatrix} 1+\cos^2\alpha_1 \\ -2\cos\alpha_1\cos\left(v_K + \arcsin\left(\sin v_K / \cos\alpha_1\right)\right) \end{bmatrix}^{0,5} / \phi^2 \qquad 6)$$

The third option, $v_K > \overline{v}_2$. In this case ΔB_2OM we have:

$$r_k = -OB_2\sin\Theta / \sin v_K \qquad (17)$$

Here, the minus sign indicates that the angle v_k is located to the right of the vertical axis of the drum (Fig. 3). However, this relation is also true for positive angles v_k. We consider here only the angles v_k, which are less than $\overline{v}_2$.

So far as $OB_2 = \left(R\cos\Theta/3\right)/\phi^2$, then

$$r_k = \left(-R\sin\Theta\cos\Theta/3\right)/\phi^2\sin v_K \qquad (18)$$

Angle [theta], necessary for calculating (18) at the predetermined is determined from the equation

$$\Theta = \cos\Theta/3\sin\left(v_K - \Theta\right) - \sin v_K \qquad (19)$$

Setting lining profile. In the proposed method we take into account any possible variants of the transverse profile of the lining. All corners of the lining profile (Fig. 4) are numbered from left to right, the extreme left point A has zero number. Corner point B_i is defined by the following coordinates: the arc length $l_{\phi i}$ and height $h_{\phi i}$. Arc $l_{\phi i}$ is taken along the circumference of the drum between her radii drawn through points A_i and B_i. With sufficient accuracy for practical calculations, the arc $l_{\phi i}$ may be replaced with corresponding chord, if the spacing of lining is sufficiently small compared with the radius of the drum. The height $h_{\phi i}$ of the point B_i is defined along the radius of the drum to the contour of circumference. The corresponding values for the point A_i are taken as follows: the length of the arc is equal to zero and the height is the height of the last corner point of the lining. Arc length of the last corner point is taken as a spacing of lining. It is necessary to pay attention to the following fact-with overlap lining the length of liners is not equal to the spacing of lining.

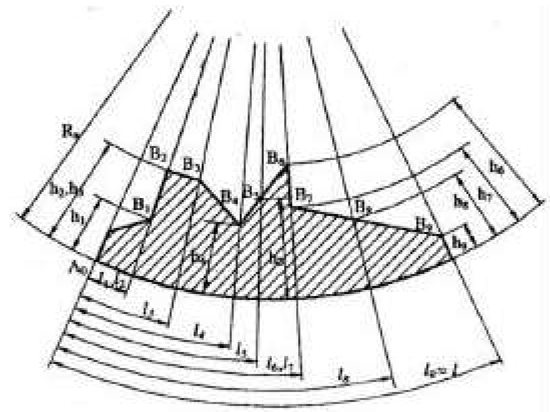

Fig. 4: The calculation scheme of lining profile

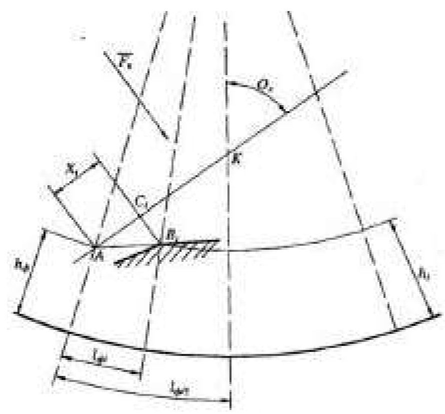

Fig. 5: To determine the coordinates of the corner points of lining

The relative coordinates of the corner points. As the relative coordinate x_i (Fig. 5) of corner point B_i, we consider the length of the segment between the point A and the perpendicular through the point B_i on a straight line passing through the point A angle [theta]$_K$ to the center line of the liner spacing and perpendicular to the direction of sector force F_K. Our goal is to obtain a formula for the calculation x_i of the known l_ϕ, [theta]$_k$, $l_{\phi i}$, $h_{\phi i}$ and h_ϕ-the height of point A.

From ΔAC_iB_i (Fig. 5) we have:

$$AB_i = \begin{bmatrix} \left(R-h_\phi\right)^2 + \left(R-h_{\phi i}\right)^2 \\ -2\left(R-h_\phi\right)\left(R-h_{\phi i}\right)\cos l_{\phi i}/R \end{bmatrix} \qquad (20)$$

So far as the range of variation of the angle OAB is (0, [pi]), then

$$\angle OAB_i = \arccos\frac{\left(R-h_\phi\right)^2 + AB_i^2 - \left(R-h_{\phi i}\right)^2}{2\left(R-h_\phi\right)AB_i} \qquad (21)$$

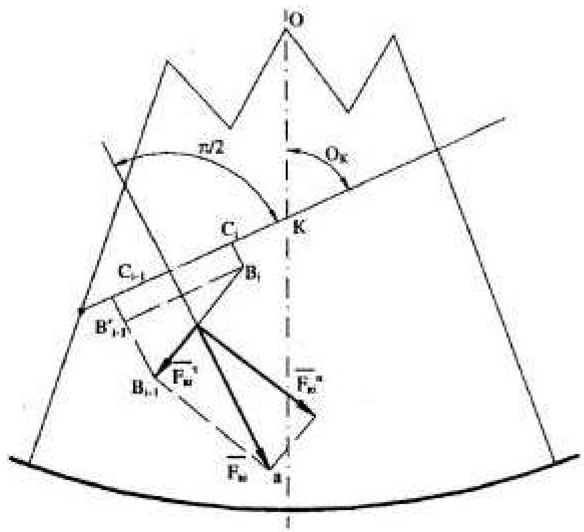

Fig. 6: Design scheme of the definition of sector forces

$$\angle OAK = \pi - l_\phi / 2R - (\pi - \Theta_k) = \Theta_k - l_\phi / 2R \tag{22}$$

Because

$$\angle C_i A B_i = \angle OAB_i - \angle OAK \tag{23}$$

then from ΔAB_iC_i we obtain:

$$x_i = AB_i \cos \angle C_i AB_i \tag{24}$$

Here AB_i-the part of lining profile, the length of which we state.

Note that if the point B_i is above AK or to the right of the center line of the sector and in all other cases the formulas (21)-(24) remain valid.

Decomposition of sector forces to the normal and tangential components. To determine the coefficient of adhesion each of the sector forces F_{Ki} should be decomposed into normal component F^n_{Ki}, directed perpendicular to the considered site of lining profile and tangential component F^τ_{Ki} (Fig. 6).

The final formula for calculating F^τ_{Ki}:

$$F^\tau_{Ki} = \text{sign}\begin{pmatrix} AB_i \sin \angle C_i AB_i \\ -AB_{i-1} \sin \angle C_{i-1} AB_{i-1} \end{pmatrix} \frac{B_{i-1}B'_i}{B_{i-1}B_i} F_{Ki} \tag{25}$$

here sign(x)-function, equal +1 if x>0 and-1 if x<0, but F^n_{Ki} is always positive and can therefore be defined as:

$$F^n_{Ki} = \left[F^2_{Ki} - \left(F^\tau_{Ki} \right)^2 \right]^{0,5} \tag{26}$$

In these calculations, we have considered the various position of section B_{i-1} B_i, from which follows that (25) and (26) are valid for all angles Θ_k and sectors of polygonal line B_{i-1} B_i, displaying any lining profile.

The calculation of the coefficient of adhesion. The final formula for the determination of the coefficient of adhesion, according to the calculation method discussed here, has the following form:

$$k_{c1} = \sum_{k=2}^{N-1} \sum_{i=1}^{n} F^\tau_{ki} \Big/ \sum_{k=2}^{N-1} \sum_{i=1}^{n} F^n_{ki} \tag{27}$$

where: N-number of sectors; n-number of corner points (polyline segments describing the profile of the liner) in one spacing of lining.

DISCUSSING THE RESULTS OF CALCULATION

In order to assess the accuracy of the developed analytical method of calculating the coefficient of adhesion of linings of different cross-section we take as an example liner profile, considered by Kryukov, DC, in calculation by their semi-graphical method [10].

As by Kryukov D.K. the load circuit is divided into 11 sectors and the transverse profile of lining is approximated by six sections of the polyline. As initial values there are taken the same: [phi] = 0,4; [phi] = 0,8; R = 1,6 m; R_1 = 1,518; k_{cM} = 0,607.

Table 1 shows the characteristics of the cross-section profile of lining under consideration.

Thus, the results of comparative calculations show that the value of the coefficient of adhesion of the profile of the lining, obtained by Kryukov D.K. with his graphical method is k_c = 0,328 and the value calculated by us (Table 2, example 1)-k_{c1} = 0,311. The discrepancy between the calculated coefficients of adhesion is about 5%. Of a more exact match we can speak only as about random one, for graphical representations are always approximate and have large inaccuracies.

Table 1:

# corner points of the profile	Coordinates of the corner points of the profile	
	Spacing, l_ϕ, cm	Height, h_ϕ,cm
1	2,69	2,91
2	4,87	3,71
3	9,45	6,33
4	16,36	8,73
5	23,85	9,60
6	41,16	9,82

Table 2: Effect of parameters R,[phi],[Phi] on the friction coefficient

			The coefficients of		
# s/p	Radius drum, m	Relative speed	Fill	Shear profile	Friction
1	1,6	0,80	0,40	0,80	0,311
2	1,6	0,80	0,40	0,60	0,295
3	1,6	0,80	0,40	0,40	0,295
4	1,6	0,80	0,40	0,40	0,297
5	1,6	0,70	0,40	0,40	0,401
6	1,6	0,70	0,30	0,40	0,462

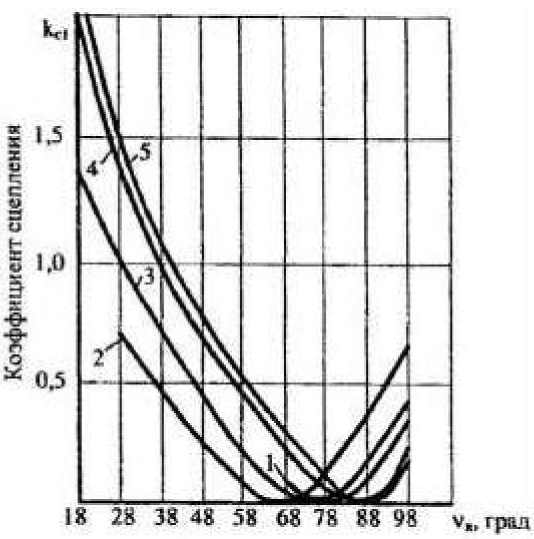

Fig. 7: Calculated relations $k_c(v_K)$

Parameter L/H: 1-1,3; 2-1,5; 3-1,9; 4-2,5; 5-4,2.

The results of calculations made on the method developed by us, are illustrated by the graphs in Fig. 7.

The nature of the curves 1-5 (Fig. 7) shows that the functions $k_{ci}(v_k;L/H)$ are of an extreme nature. The highest value of the coefficient of adhesion boot liners are in the range of angles $18° < v_k < 38°$, here $k_c < 1,0$. With an increase in the ratio L/H, i.e. increasing step between the projections, while increasing the height of projections, the friction coefficient also increases. This download is slipping relative to the lining.

In the angles characterizing the loading position, limited $38° < v_k < 110°$, friction coefficient $k_c < 1,0$. Moreover, in the area of sector angle $68° < v_k < 88°$ $k_{c1} \approx 0$- here about download liner slip, because $k_{c1} < k_T$. However, for each of the parameter L/H the minimum of function $k_{c1}(v_k)$ is depended of its inherent point. For example, the profile of the liner portion when L/H = 1,3 k_{c1} = min in this case $v_k = 80°$, when L/H = 1,5; k_{c1} = min, but already at $v_k = 68°$. That is, depending on the ratio L/H, characterizing the profile lining MT relative slippage occurs lining in different areas of boot.

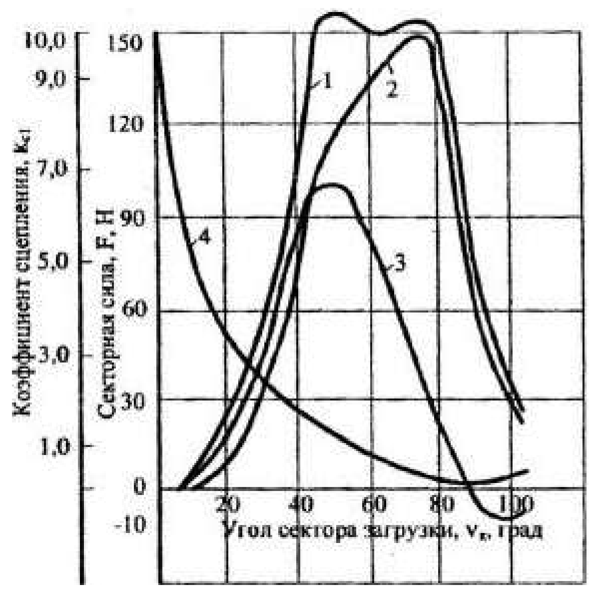

Fig. 8: Calculated dependence to k_c, $F(v_k)$
1-sector force, F_k; 2-normal; 3-tangential components of the sector strength; 4-coefficient of friction

The maximum lining life achieved in those cases where there is no slippage of grinding media. This is possible with $k_c \le k_T$. On the basis of Fig. 7, the mode of grinding media is observed in the area of cross-circuit at load $48° < v_k < 68°$, i.e. in the middle part of the circuit load is evident. However, any ratio L/H, i.e. in all areas along the profile plate slips relative to none of the lining in the sector of angles $68° < v_k < 88°$.

This conclusion is confirmed by the nature of the forces of change in the sector, ie, pressure on the lining of the grinding bodies (Fig. 8).

The greatest amount of sector force F_k ?????????, reaches for any L/H in the sector of angles $48° < v_k < 75°$. This area is characterized by the growth $F_k(v_k)$ and decay $F_k^{[tau]}(v_k)$. When $v_k > 88°$ $F_k^{[tau]}(v_k)$ is negative, ie changes direction (Fig. 8, 3). In this zone, the circuit load with the greatest force presses the lining because

$k_c \approx 0$. From (25) it follows that the maximum value k_c is provided at $F_k^{[tau]} \rightarrow \max$ and $F_k^n \rightarrow \min$, or when (L/H)$\rightarrow$min, but with the proviso that L = const and H$\rightarrow$max. However, the required reduction ratio L/H is possible only by increasing the height of the projections lifters on the surface of the lining, which considerably affects the behavior of the grinding bodies and an excessive increase in the height of the projections causes the balls are being moved through the "heel" on the lining, without performing work grinding. In this connection the question of the rational profile liner in particular shelf is very important, hitherto under-researched.

The calculation of the coefficient of friction for longitudinal movement of load. Let us find the form of the function k_{c2}([xi],[betta],[epsilon]).

In mills with transverse movement of the longitudinal load level during the cycle is changed to a sufficiently large height. Since k_C the amount of influence [phi] and in mills with transverse longitudinal movement relative load factor and load level change with angle, the phase position of the drum and the angle of repose of the load, the friction coefficient and the value is changed with the changeand in mills with transverse longitudinal movement relative load factor and load level change with angle, the phase position of the drum and the angle of repose of the load, the friction coefficient and the value is changed with the change [xi],[betta],[epsilon].

Therefore, cross-circuit load in mills with a cross-longitudinal motion of grinding bodies should take into account the angle of repose of the boot.

In accordance with the theory of the Corner, the curve of internal cross-circuit track download is part of a logarithmic spiral, with a pole at the point M (Fig. 9), described by the equation:

$$r_k = ce^{\varepsilon\tau} \qquad (28)$$

where c = v/ω coefficient taking into account the rate of change of the radius r.
From Fig. 9 it follows that (MO = g/ω^2)

$$r_k = g/\omega^2 + R - h/\cos\varepsilon \qquad (29)$$

ie r_k(h), a h([xi],[betta],[epsilon]), then r_k([xi],[betta], [epsilon]).

In drum-type mills, equipped with an inclined wall velocity of the longitudinal motion of grinding media is:

$$V_{\Pi p} = 2\phi\left(gR\right)^{0,5} ctg\beta\cos\xi ctg\varepsilon/\pi \qquad (30)$$

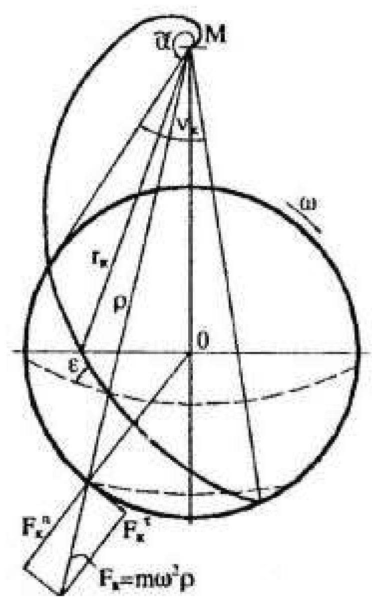

Fig. 9: Design model of the coefficient of friction in a changing load level

and the speed of the transverse motion of grinding media is:

$$V_{BEP} = 2\phi(gR)^{0.5} ctg\beta \cdot \cos\xi \cdot tg\varepsilon/\pi \qquad (31)$$

Based on (31) the parameter C in equation logarithmic spiral applied to our problem is defined as:

$$C = 2\phi(gR)^{0.5} ctg\beta\cos\xi \cdot tg\varepsilon/\pi\omega \qquad (32)$$

Since we have developed a method of calculating the coefficient of friction, the circuit load is divided into sectors with an angle Δv_k and a pitch equal to the pitch profile liners inner radius of the loop load must be calculated taking into account the change in the pitch angle of the sector, ie, with (28) and (32) we obtain

$$r_k = 2\phi(gR)^{0.5} ctg\beta\cos\xi \cdot tg\varepsilon \cdot e^{\varepsilon\left(\overline{a}+\Delta v_k\right)}/\pi\omega \qquad (33)$$

The equation (33) for calculating the radius of the inner contour load allows an inclination angle partitions ([betta]) angle walls ([xi]) and the angle of repose of the load ([epsilon]) and the frequency of rotation of the drum mill.

In determining r_k the parameter C, in equation logarithmic spiral (28) adopted by us in the classical formulation of the problem, i.e. accepted that c = V/[omega]. If the velocity V_{np} of the longitudinal displacement of a point along the radius vector r_k is the rate of decrease loading height, the angular velocity of the radius vector r_k is not the angular velocity of the drum mill. Furthermore, in equation (33) for calculation r_k we must know the boundary of angles $\tilde{a}$.

Based on the design scheme (Fig. 9), we write the equation of a circle (drum mills) in polar coordinates:

$$r_k^2 - 2r_k \cos\tilde{a}/\phi^2 + \phi^{-4} - 1 = 0 \quad (34)$$

The roots of equation (34) are

$$r_k = -\phi^{-4}\sin\tilde{a} \pm \left[\phi^4 \sin^2\alpha - \phi^4 + R^2\right]^{0.5} \quad (35)$$

and

$$\cos^2\tilde{\alpha} \le R^2\phi^2 = \phi^4$$

i.e. $\arccos(-\phi^2) \le \tilde{a} \le \arccos\phi^2$.

Equation (28) can also be represented as:

$$\tilde{a} = \varepsilon^{-1}\ln r/c \quad (36)$$

Then, by (36), the equation of the circle becomes

$$r_k^2 + 2\phi^{-2}\sin\left(\varepsilon^{-1}\ln r_k/c\right)r_k + \phi^{-4} - R^2 = 0 \quad (37)$$

The roots of this equation are the limiting values of the inner radius of the circuit load.

To understand the physical nature of the parameter C in the annex to our problem, we assume that the original equation logarithmic spiral angle [alpha] has a different value, for example [alpha]′ = [alpha]+Δ[alpha], then the parameter C in equation (28) will be different in magnitude and self equation becomes:

$$r_k = c'e^{\varepsilon\tilde{\alpha}'} = c'e^{\varepsilon(\tilde{\alpha}+\Delta\alpha)} = c'e^{\varepsilon\Delta\tilde{\alpha}} \quad (38)$$

Equating (28) and (38) we have:

$$c = c'e\varepsilon\Delta\tilde{\alpha} \quad (39)$$

On the basis of (28), (38), (39) it is easy to show that we are interested in the solution space of (28) is limited to the parameter value C and [alpha] which are respectively:

$$\phi^{-2} \le c \le \phi^{-2} + 1;\ -0.5\pi \le \tilde{\alpha} \le 0.5\pi \quad (40)$$

Solving equation (28) and (34) we obtain an equation for calculating the limiting values of the angle [alpha], i.e. $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ $\left(\tilde{\alpha}_1 < 0; \tilde{\alpha}_2 > 0\right)$:

$$c^2e^{2\varepsilon\tilde{\alpha}} - 2c\cos\tilde{\alpha}e^{\varepsilon\tilde{\alpha}}/\phi^2 + \phi^4 - 1 = 0 \quad (41)$$

To determine the relative load factor will calculate the area of a circle which cut off the logarithmic spiral

$$S = \iint r\dot{r}\dot{\alpha} = \int_{\tilde{\alpha}_1}^{\tilde{\alpha}_2}\dot{\alpha}\int_{r=ce^{\varepsilon\tilde{\alpha}}}^{r=f(\tilde{\alpha})} r\dot{r} \quad (42)$$

where $r - f(\tilde{\alpha})$ -the equation of a circle in polar coordinates

The relative load factor, taking into account (42), is equal to:

$$\phi' = 0{,}25\pi^{-1}[2\tilde{\alpha} + \sin2\tilde{\alpha}\phi^{-4} - \varepsilon^{-1}c^2e^{2\varepsilon\tilde{\alpha}} + 2\phi^{-2}\sin\tilde{\alpha}\,*$$
$$*(1-\phi^{-4}\sin^2\tilde{\alpha})^{0,5} + 2\arcsin\phi^{-2}\sin\tilde{\alpha}]_{\tilde{\alpha}_1}^{\tilde{\alpha}_2} \quad (43)$$

Thus, we have derived an equation to calculate the length of the inner radius of the load r_k, which takes into account all the factors that determine the nature and dynamics of transverse-longitudinal motion download r_k ([betta],[epsilon],[Phi],[alpha]).

The subsequent calculation k_{c2} is to ensure that in determining the strength of sector we expect the inner radius of the circuit load by (33).

CONCLUSION

The combination of mutual arrangement of elements allows the liner to receive any lining, including variables, the coefficient of adhesion in both the longitudinal and transverse sectional mill drum.

The calculations show that with the increase in the diameter of the drum rotation speed and load factor-the friction coefficient decreases (slip grinding media regarding the lining decreases), increasing the height of the projections, at the same step, decreases k_c, with smooth lining even a significant change D, [phi], [Phi] in the value of the coefficient of friction changes slightly.

Conclusions. Analysis calculations show that during the cycle the value of friction coefficient changes from lowest to highest according to a sinusoidal law. At the same profile linings and other things being equal ([phi], [betta],[epsilon],[Phi]) = const k_{c2} greater at higher relative load factor and less with less [Phi]′. To stabilize the coefficient of coupling, for example, using liner of the rolling elements is necessary that the height of the projections in the long part of the chamber was greater than short. Profile lining determines not only the magnitude k_c but also the dynamic and energetic parameters of the load.

In this regard, we defined according to analytical geometry influence the profile of the lining on the operation of grinding media can reveal more fully the impact of the construction inner mill energy-exchange devices for process and energy performance of the grinding bodies and grinding process in drum mills in general.

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