

Determining Average Velocity of Small Rivers of Kazakhstan

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Abstract: The Republic of Kazakhstan has about 39,000 rivers, including 15 large rivers and 40 medium. The rest, i.e. more than 90% of the rivers, can be referred to as small rivers with shallow depths. To date, all the methods of average flow rate calculation have been designed for medium and large rivers and cannot be used for small streams. Therefore this article offers the formula for determination of the average rate in small rivers of Kazakhstan. But currently, the most widely used formulas for finding the average rate on a speed vertical are based on the approximate method of dividing the rate curve into the equal parts. This approach reduces the accuracy of the average rate estimation at finding the water flow rate. Therefore, the aim of the study was to develop a better method for calculation of the water flow rate which would allow measurements at shallow depths using a mathematical expression that would describe the distribution of the longitudinal rate at different depths along the vertical more accurately. The research has shown that the distribution of the longitudinal rate at different depths is fairly described by the parabola equation that served a basis for the developed formula for calculation of the average rate on the speed vertical. The experimental, theoretical and field studies have shown the results that were 8-9% more consistent, which is quite acceptable in water management and water and energy calculations.

Key words: Rate curve • Average rate • Average rate of water flow • Speed vertical

INTRODUCTION

If the cross section of the flow is perpendicular to the velocity direction and coincides with an effective cross-section of the flow, the inflow rate in the reservoir is determined by the formula [1, 2], etc.

$$Q = \int_0^w v dw \quad (1)$$

where v is flow velocity within the elementary area and dw - the value of the elementary area.

Based on the method of determining the inflow rate in the reservoir, it is necessary to specifically identify the location of the effective cross-section of the stream so that in all elementary areas the velocity vector is directed to the standards, then the system (1) can be written in the following form $dw = dhdb$:

$$Q = \int_0^b \int_0^h v dh db = \int_0^b q db \quad (2)$$

Next, for direct determination of the inflow rate in the reservoir the kinematic and geometric elements of the flow should be measured. To do this, approximate the integral (2) and write the equation as follows:

$$Q = kq_1b_1 + \frac{q_1 + q_2}{2}b_2 + \dots + \frac{q_{n-1} + q_n}{2}b_n + kq_nb_{n+1} \quad (3)$$

where q_1, q_2, \dots, q_n are water flow rates on the verticals, m^3/s ; $b_1, b_2, \dots, b_n, b_{n+1}$ - the distance between the vertical lines, m; k - coefficient for the rates on the coastal verticals taken as 0.7 at sloping bank with $h = 0$ on the shore line; 0.8 - at the steep bank of the river or uneven wall of the channel; 0.9 - at a smooth concrete wall of the channel [1, 3, 4].

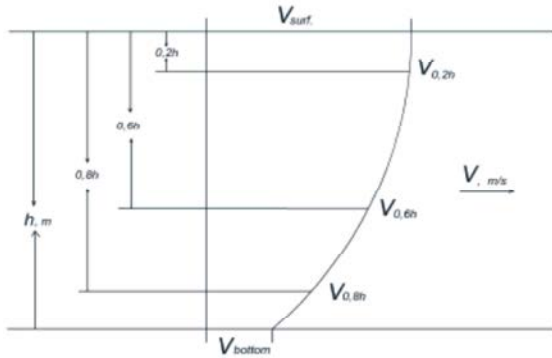


Fig. 1: Distribution of longitudinal velocities at different depths over vertical.

The flow rates on the verticals are calculated on the following formula:

$$q = V_{average} h \quad (4)$$

where V_{av} is the average velocity on the vertical lines, m/s; h - the depth of individual verticals, m.

One of the most critical and complex challenges in determining the elementary flow rates (water flow rates on the verticals) is the average velocity on the vertical lines. Distribution of water flow rates in open channel flows can be quite diverse. It depends on the type of supply of the river, the morphological features, the bed roughness and slope of the water surface [4-6].

Consider the propagation of longitudinal velocities at different depths over the vertical. Lay off the velocity values from the direction of vertical and connect their ends by the smooth line; this line will be the velocity profile (Figure 1).

Study: For the mathematical expression of the velocity profile different authors proposed numerous formulas at different times, in particular the equation of parabola, hyperbola, logarithmic curve, etc. One of the first, who used the equation of a parabola, was I.A. Girillovich [7-9]; below is this equation.

$$V = V_{surf} \sqrt[n]{1 - \frac{y}{h}} \quad (5)$$

where V is water velocity in the characteristic points, m/s;

V_{surf} - Surface velocity, m/s;

y - Distance from the water surface to a point with velocity, m/s;

h - The depth of the vertical, m/s.

Until now, it has been widely spread in water management and hydropower practice by means of splitting the profile area with horizontal lines into parts, which can be approximately considered as trapeziums (Figure 1). In this case, the flow rate can be defined as a sum of the areas of the elementary trapeziums [5, 7, 8]:

$$Q = 0,5 (V_{surf} + V_{0,2h}) 0,2h + 0,5 (V_{0,2h} + V_{0,6h}) 0,4h + 0,5 (V_{0,6h} + V_{0,8h}) 0,2h + 0,5 (V_{0,8h} + V_{bot}) 0,2h \quad (6)$$

After reduction of similar terms and division by the depth h , obtain the averaged value of stream:

$$V_{average} = 0,1 (V_{surf} + 3V_{0,2h} + 3V_{0,6h} + 2V_{av} + V_{bot}) \quad (7)$$

where $V_{0,2h}$ is the flow velocity in the point $0.2h$ from the water surface, m/s;

$V_{0,6h}$ = The flow rate in the point $0.6 h$ from the water surface, m/s;

$V_{0,8h}$ = The flow rate at $0.8 h$ from the water surface, m/s;

V_{bot} = Bottom velocity, m/s.

Velocities at a depth of $0.2 h$, $0.6 h$ and $0.8 h$ are given weights (coefficients).

Thus, the above formula (7) was obtained by summing the areas of the four trapeziums only with different weighting coefficients, which shows low accuracy.

As it is shown above, the flow rate distribution on the vertical (velocity profile) can be obtained based on the actual measured data. It is often necessary to obtain the velocity distribution in the points over the depth and the average velocity on the vertical in the absence of detailed measurements, based on the theoretical constructs. In this case, the initial data are: maximum speed, channel roughness and slope of the river. The use of such dependencies is of great practical importance, since the method of determining the Chezy coefficient was developed rather fully and the value of the surface velocity is not difficult to determine in practice without making detailed measurements over the depth [10-12].

G.V. Zheleznyakov [5, 10] in his work widely used the equation of parabola to describe the longitudinal velocity at different depths over the vertical; this equation is expressed as follows.

$$V = V_{surf} \left(\frac{y}{h} \right)^{\frac{1}{m}} \quad (8)$$

where V_{surf} - surface velocity, m/s

$$m = 0.35C + 3 \quad (16)$$

h = Vertical depth, m

y = Distance from the water bottom to the point with velocity V ,

m = An empirical coefficient, which is determined by:

$$m = \frac{C_B}{\sqrt{g}} \left(\frac{2\sqrt{g}}{\sqrt{g} + C_V} + 0.30 \right) \quad (9)$$

where C_v - Chezy coefficient on the vertical $m^{0.5}/s$;

g - acceleration of gravity, m/s.

Then, the average water flow rate on the vertical is calculated [13-16]:

$$V_{average} = V_{surf} \left(\frac{m}{1+m} \right) \quad (10)$$

In addition, in water management practice the equation of ellipse is used for the mathematical expression of the line of velocity profile, derived by A.V. Karaushev [13].

$$V_{average} = V_{surf} \sqrt{1 - P \left(\frac{y}{h} \right)^2} \quad (11)$$

where P is dimensionless parameter whose value depends on the Chezy coefficient;

y - the distance from the point of determination of velocity from the surface, m.

In A.V. Karaushev's work [13], to calculate the average velocity on the vertical the following is proposed.

$$V_{average} = \sqrt{\frac{PCV_{surf}^2}{M}} \quad (12)$$

where C - Chezy coefficient, $m^{0.5}/s$;

M - function of Chezy coefficient, $m^{0.5}/s$.

Chezy coefficient function is set:

$$\text{at } 10 \leq C \leq 60 \quad M = 0.7C + 6 \quad (13)$$

$$\text{at } C > 60 \quad M = 48 \text{ const} \quad (14)$$

Another formula of A.V. Karaushev [13] is:

$$V_{average} = \frac{V_{surf}}{1+m3.c} \quad (15)$$

where m is empirical coefficient which is calculated:

And finally, the formula of A.V. Karaushev [13] with a simpler form is:

$$V_{average} = \frac{V_{surf}(C-1)}{1,11.C} \quad (17)$$

The disadvantages of these formulas are determination of the average velocity over the vertical on one measured velocity (V_{surf}) and inability to accurately determine the roughness of the river bed [1, 17, 18].

In this connection there was a need to develop an approach that would provide sufficient accuracy at other conditions being equal. As it was repeatedly noted above, for the mathematical expression of the longitudinal velocity distribution at various depths the most commonly used is the parabola equation of different degrees with the horizontal and vertical axes.

Further, on the well-known method the velocities of water flow in the points [1, 17, 18] were determined. According to the measured velocities in the points on each speed vertical the velocity profiles were built, for which measurement points were laid off at certain depths, the ends of the vectors were denoted by asterisks and the theoretical curve corresponding to the shape of the velocity profile was plotted (Figure 2 a, b, c, d).

The characteristic profiles on Figure 2 a, b, c and d were obtained from the results of long-term field studies on the river Talas and the bypass channel (concrete channel, $b = 10m$) below the Talas dam on the specially equipped hydrometric bridges.

The resulting characteristic profiles of different measurements in the same alignment in different years on the river Talas correspond to the equation of parabola of 3-5 degree (Figure 2 a, b) and on the bypass channel correspond to the equation of parabola of 8-10 degree (Figure 2 c, d). The feature of the profiles in the bypass channel is a slight difference between surface and bottom velocities associated with the bottom roughness of artificial structures.

Therefore, knowing in advance the shape of the velocity distribution curve, that is, the equation of parabolic trapeziums, one can write the total area of the curvilinear trapezium [17, 19]:

$$\int_a^b y dz = \frac{h}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + 2y_{n-2} + 4y_{n-1} + y_n) \quad (18)$$

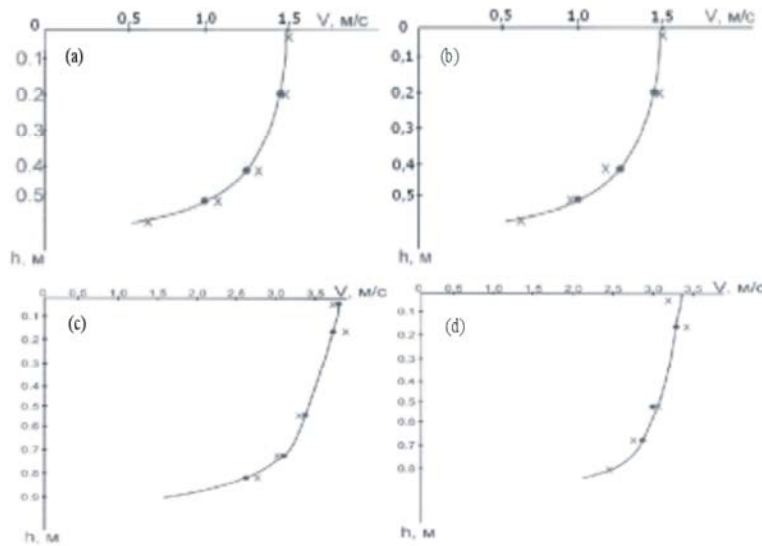


Fig. 2: Characteristic profiles of velocity distribution over the vertical in the river Talas and the bypass channel (a, b - river Talas, c, d - a bypass channel); x - measured points ; o - theoretical points.

Considering the above equation for five ordinates (if $n = 2$; $h = \frac{b-a}{n}$) and dividing it by the length of equal

parts, obtain an approximate formula for determining the center line of the curvilinear trapezium, i.e.:

$$y_{av} = \frac{1}{6}(y_0 + 4y_1 + y_2) \quad (19)$$

Having transformed the equation (19) with respect to the velocity profile with a vertical axis, obtain [13, 14, 19]:

$$V_{average} = \frac{1}{6}(V_{surf} + 4 \cdot V_{av}^{con} + V_{bot}) \quad (20)$$

where V_{bot} - bottom velocity, m/s and

V_{av}^{con} - Conditional average velocity of a parabolic trapezium, m/s.

The conditional average velocity of the parabolic trapezium on the speed vertical is proposed to calculate as follows.

$$V_{av}^{con} = \sqrt{\frac{V_{surf}^2 + V_{bot}^2}{2}} \quad (21)$$

Next, substituting equation (21) into (20) we finally obtain the average velocity of the parabolic trapezium with a vertical axis:

$$V_{average} = \frac{1}{6} \left(V_{surf} + 2\sqrt{2(V_{surf}^2 + V_{bot}^2)} + V_{bot} \right) \quad (22)$$

Summing up, it may be noted that for the mathematical expression of the line of the velocity profile at different depths over the vertical both in natural flow and in artificial structures the preference is given to the equation of the parabola with different degrees.

The results of comparative calculations between the existing and the proposed formulas are given in Table 1.

As it can be seen from Table 1, the deviations of the calculated values do not exceed 2... 3 %, which is quite acceptable in the water management calculations.

Findings:

- As a result of field studies it was found that the distribution of longitudinal velocities at different depths over the vertical in open channel flows is well described by the equation of parabolic trapezium of different degree.
- The profile of the flow velocity in the channel of the river Talas corresponds to the equation of a parabolic trapezium with an exponent $n = 3-5$ (Figure 2 - a, b) and the velocity profile in the bypass channel (artificial structure) is described by a parabolic trapezium with the exponent $n = 8-10$ (Figure 2 - c, d), which is confirmed by the theoretical studies of N.A. Girillovich, G.V. Zheleznyakov, *et al.*

Table 1: Comparisons of average velocities on the vertical, calculated by different formulas and the proposed formula.

N	Authors	Formulas	Average velocities m/s	% of deviation
1	2	3	4	5
1	N.A. Girillovich	$V_{av} = 0,1(V_{surf} + 3.V_{0,2} + 3.V_{0,6} + 2.V_{0,8} + V_{bot})$	0.0	0.00
2	G.V. Zheleznyakov	$V_{av} = V_{surf} \left(\frac{m}{1+m} \right)$	0.68	2.86
3	A.V. Karaushev	a $V_{av} = V_{surf} \sqrt{1 - P \left(\frac{y}{h} \right)^2}$	0.69	1.43
		b $V_{av} = \sqrt{\frac{V_{surf}^2 \cdot PC}{M}}$	0.68	2.86
		c $V_{av} = \frac{V_{surf}}{1+m3c}$	0,68	2,86
		d $V_{av} = \frac{V_{surf}(C-1)}{1,11.C}$	0.69	1.43
4	Proposed formula	$V_{av} = \frac{1}{6} \left(V_{surf} + 2\sqrt{2(V_{surf}^2 + V_{bot}^2)} + V_{bot} \right)$	0.68	2.86

- A formula for determining the center line of the parabolic trapezium corresponding to the average velocity of the water flow on the speed vertical is proposed.
- The results of comparative calculations of the field and theoretical studies using the proposed and existing formulas give similar results and do not exceed 10%, which is quite acceptable in water management and water and energy calculations.

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