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Application of Quintic B-Spline Collocation Method for Solving the Coupled-BBM System

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Abstract: In this work, Quintic B-spline collocation technique for the coupled BBM-system of Boussinesq type has been presented. The technique is based on the Crank-Nicolson formulation for time integration and quintic B-spline functions for space integration. The accuracy of the proposed method is illustrated by studying a solitary wave motion. The interaction of two solitary waves is used to discuss the effect of the behavior of the solitary waves after the interaction . The results are presented and compared against analytic solution of the system.

Key words: Coupled BBM-system · Solitary waves · Quintic B-spline

INTRODUCTION

In this paper, we consider the Coupled-BBM system, which belongs to the class of Boussinesq systems, modeling two-way propagation of long waves of small amplitude on the surface of water in a channel. The system is a good candidate for modeling long waves of small to moderate amplitude. The Coupled BBM-system is given by Bona and Chen [1],

$$\begin{aligned} &v_t + u_x + (vu)_x - \frac{1}{6} v_{xxt} = 0 \\ &u_t + v_x + uu_x - \frac{1}{6} u_{xxt} = 0 \end{aligned} \tag{1}$$

where x corresponds to distance along the channel and t is the elapsed time, $v(x, t)$ is a dimensionless deviation of the water surface from its undisturbed position and $u(x, t)$ is the dimensionless horizontal velocity above the bottom of the channel.

The boundary conditions are chosen from:

$$\begin{aligned} &v(0, t) = \alpha_1,\ v(L, t) = \alpha_2,\ u(0, t) = \beta_1,\ u(L, t) = \beta_2 \\ &v_x(0, t) = 0,\ v_x(L, t) = 0,\ u_x(0, t) = 0,\ u_x(L, t) = 0 \end{aligned} \tag{2}$$

and the initial conditions are

$$v(x, 0) = f(x),\ u(x, 0) = g(x) \tag{3}$$

The theoretical results like existence of line solitary waves, line cnoidal waves symmetric and asymmetric periodic wave pattern have been discussed in [2-5]. We refer the reader to Chen *et al*. [6] who derived the existence of periodic traveling-wave solutions $(v(x, t), u(x, t))$ the form.

$$v(x,t) = v(x - wt) = \sum_{n=-\infty}^{\infty} v_n e^{i(n\pi/l)(x-wt)}$$

$$u(x,t) = u(x - wt) = \sum_{n=-\infty}^{\infty} u_n e^{i(n\pi/l)(x-wt)}$$

where l and w connote the half-period and the phase speed, respectively.

Rigorous errors estimate for Bona-Smith and Coupled-BBM type systems were proved in [7]. The solution of (1) approximates the solution of Euler's equation with the order of accuracy of the equation, namely, for any initial value $(v_0, u_0) \in H^\sigma(\Re)^2$ with $\sigma \geq s \geq 0$ large enough, there exists a unique solution (v, u) of Euler equations, such that, Bona *et al*. [8].

$$\|v - v_{euler}\|_{L^\infty(0,t;H^s)} + \|u - u_{euler}\|_{L^\infty(0,t;,H^s)} = o(\varepsilon_1^2 t, \varepsilon_2^2 t, \varepsilon_1\varepsilon_2 t) \text{ for } 0 \leq t \leq o(\varepsilon_1^{-1}, \varepsilon_2^{-1})$$

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One of the advantages that (1) has over alternative Boussinesq-type systems is the easiness with which it may be integrated numerically [9]. Furthermore, it was proved in [9, 10] that the initial value problem either for $x \in \Re$ or with boundary conditions ($x \in [a, b]$) for (1) is well posed in certain natural function classes.

The initial-boundary value problem of the form (1) posed on a bounded smooth plane domain with homogenous Dirichlet or Neumann or reflective (mixed) boundary conditions which is locally well-posed [11].

The existence and uniqueness of the system have been proved in Bona *et al*. [10]. They investigated the solution of the system as integral equation, while Chen in [12] established the existence of solitary waves for several Boussinesq types, including the Coupled-BBM system.

Various numerical techniques including the finite element method have been used for the solution of Bona-Smith system of Boussinesq type in Antonopoulos *et al*. [13]. Numerical schemes using B-spline methods have been successfully applied to solve various nonlinear partial differential equations. For instance, a numerical solution based on the collocation method with quintic B-spline function was set up to obtain the solution of Korteweg-de Vries Burgers equation by El-Danaf [14] and Extended Fisher-Kolmogorov equation by Mittal and Arora [15]. Soliman and Raslan in [16] solved RLW equation by collocation method using quadratic B-spline as element shape function.

The structure of the paper is as follows. In section 2, quintic B-spline collocation method is designed for the numerical solution of the Coupled BBM-system. In section 3, the method is applied to the time-split Coupled BBM-system. The results and discussions are presented in section 4. In the last section, a summary of the main conclusions is given at the end of the paper.

Quintic B-Spline Collocation Method: Consider a mesh $a = x_0 \langle x_1 \langle x_2 \langle \cdots \langle x_n = b$ as a uniform partition of the solution domain $a \le x \le b$, with $h = x_i - x_{i-1}$, $i = 1,2,,,,, N$. Our numerical treatment for solving equation(1) using collocation method with quintic B-spline function is to find an approximate solution $U_N(x, t)$, $V_N(x, t)$ to the exact solution $u(x, t)$, $v(x, t)$ in the form:

$$\begin{aligned} U_N(x,t) &= \sum_{i=-2}^{N+2} \delta_i(t) B_i(x) \\ V_N(x,t) &= \sum_{i=-2}^{N+2} \sigma_i(t) B_i(x) \end{aligned} \tag{4}$$

where δ_i and σ_i are time dependent quantities to be determined from the collocation form of the Coupled BBM-system. The Quintic B-spline $B_i(x)$ at the notes x_i defined by:

$$B_i(x) = \frac{1}{h^5} \begin{cases} (x-x_{i-3})^5 & [x_{i-3}, x_{i-2}] \\ (x-x_{i-3})^5 - 6(x-x_{i-2})^5 & [x_{i-2}, x_{i-1}] \\ (x-x_{i-3})^5 - 6(x-x_{i-2})^5 + 15(x-x_{i-1})^5 & [x_{i-1}, x_i] \\ (x-x_{i-3})^5 - 6(x-x_{i-2})^5 + 15(x-x_{i-1})^5 & \\ -20(x-x_i)^5 & [x_i, x_{i+1}] \\ (x-x_{i-3})^5 - 6(x-x_{i-2})^5 + 15(x-x_{i-1})^5 & \\ -20(x-x_i)^5 + 15(x-x_{i+1})^5 & [x_{i+1}, x_{i+2}] \\ (x-x_{i-3})^5 - 6(x-x_{i-2})^5 + 15(x-x_{i-1})^5 & \\ -20(x-x_i)^5 + 15(x-x_{i+1})^5 - 6(x-x_{i+2})^5 & [x_{i+2}, x_{i+3}] \\ 0 & \textit{otherwise} \end{cases} \tag{5}$$

where $\{B_{-2}, B_{-1}, B_0, B_1, ..., B_{N+1}, B_{N+2}\}$ forms a basis over the interval $[a, b]$ [17]. The values of $B_i(x)$ and its derivatives are tabulated in Table 1.

Numerical Solution of Coupled BBM-System Using Collocation Quintic B-Spline Method: Discritization of (1) is carried out by interpolating u, u_x, u_{xx}, v, v_x, v_{xx} using Crank- Nicolson rule and the usual finite difference method for time derivatives.

Table 1: Values of $B_i(x)$ and its derivatives at the knots points

x	x_{i-3}	x_{i-2}	x_{i-1}	x_i	x_{i+1}	x_{i+2}	x_{i+3}
$B_i(x)$	0	1	26	66	26	1	0
$B'_i(x)$	0	$\frac{5}{h}$	$\frac{50}{h}$	0	$\frac{-50}{h}$	$\frac{-5}{h}$	0
$B''_i(x)$	0	$\frac{20}{h^2}$	$\frac{40}{h^2}$	$\frac{-120}{h^2}$	$\frac{40}{h^2}$	$\frac{20}{h^2}$	0

$$
\begin{aligned}
&u_t = \frac{u^{n+1} - u^n}{\Delta t}, \quad v_t = \frac{v^{n+1} - v^n}{\Delta t} \\
&u_x = \frac{u_x^{n+1} + u_x^n}{2}, \quad v_x = \frac{v_x^{n+1} + v_x^n}{2} \\
&u_{xx} = \frac{u_{xx}^{n+1} + u_{xx}^n}{2}, \; v_{xx} = \frac{v_{xx}^{n+1} + v_{xx}^n}{2}
\end{aligned} \tag{6}
$$

Taking $\Delta t = k$, then equation (1) becomes,

$$
\begin{aligned}
&\frac{u^{n+1} - u^n}{k} - \frac{1}{6}\left(\frac{u_{xx}^{n+1} - u_{xx}^n}{k}\right) + \frac{v_x^{n+1} + v_x^n}{2} + u^n\left(\frac{u_x^{n+1} + u_x^n}{2}\right) = 0 \\
&\frac{v^{n+1} - v^n}{k} + \frac{u_x^{n+1} + u_x^n}{2} - \frac{1}{6}\left(\frac{v_{xx}^{n+1} - v_{xx}^n}{k}\right) + u^n\left(\frac{v_x^{n+1} + v_x^n}{2}\right) + \\
&v^n\left(\frac{u_x^{n+1} + u_x^n}{2}\right) = 0
\end{aligned}
$$

Last equation can be written in the form:

$$
\begin{aligned}
&u^{n+1} + \frac{k}{2}u^n u_x^{n+1} - \frac{1}{6}u_{xx}^{n+1} + \frac{k}{2}v_x^{n+1} = \\
&u^n - \frac{k}{2}v_x^n - \frac{k}{2}u_x^n u^n - \frac{1}{6}u_{xx}^n \\
&\left(\frac{k}{2} + \frac{k}{2}v^n\right)u_x^{n+1} + v^{n+1} + \frac{k}{2}u^n v_x^{n+1} - \frac{1}{6}v_{xx}^{n+1} = \\
&v^n - \frac{1}{6}v_{xx}^n - \frac{k}{2}u_x^n v^n - \frac{k}{2}u_x^n - \frac{k}{2}v_x^n u^n
\end{aligned} \tag{7}
$$

After using (4) with the values given in the table 1, we get

$$
\begin{aligned}
&(\delta_{i-2}^{n+1} + 26\delta_{i-1}^{n+1} + 66\delta_i^{n+1} + 26\delta_{i+1}^{n+1} + \delta_{i+2}^{n+1}) + \\
&\frac{k}{2}u^n\left(\frac{5}{h}(\delta_{i-2}^{n+1} + 10\delta_{i-1}^{n+1} - 10\delta_{i+1}^{n+1} - \delta_{i+2}^{n+2}\right) - \\
&\frac{1}{6}\left(\frac{20}{h^2}(\delta_{i-2}^{n+1} + 2\delta_{i-1}^{n+1} - 6\delta_i^{n+1} + 2\delta_{i+1}^{n+1} + \delta_{i+2}^{n+1})\right) + \\
&\frac{k}{2}\left(\frac{5}{h}(\sigma_{i-2}^{n+1} + 10\sigma_{i-1}^{n+1} - 10\sigma_{i+1}^{n+1} - \sigma_{i+2}^{n+2})\right) = \eta_i
\end{aligned}
$$

$$(\frac{k}{4}+\frac{k}{2}v^n)(\frac{5}{h}(\delta_{i-2}^{n+1}+10\delta_{i-1}^{n+1}-10\delta_{i+1}^{n+1}-\delta_{i+2}^{n+2}))$$
$$+(\sigma_{i-2}^{n+1}+26\sigma_{i-1}^{n+1}+66\sigma_{i}^{n+1}+26\sigma_{i+1}^{n+1}+\sigma_{i+2}^{n+1})+$$
$$\frac{k}{2}u^n(\frac{5}{h}(\sigma_{i-2}^{n+1}+10\sigma_{i-1}^{n+1}-10\sigma_{i+1}^{n+1}-\sigma_{i+2}^{n+2}))$$
$$-\frac{1}{6}(\frac{20}{h^2}(\sigma_{i-2}^{n+1}+2\sigma_{i-1}^{n+1}-6\sigma_{i}^{n+1}+2\sigma_{i+1}^{n+1}+\sigma_{i+2}^{n+1}))=\mu_i\,,\quad i=0,1,\ldots,N \tag{8}$$

where,

$$\eta_i=u^n-\frac{k}{2}v_x^n-\frac{k}{2}u_x^nu^n-\frac{1}{6}u_{xx}^n$$
$$\mu_i=v^n-\frac{1}{6}v_{xx}^n-\frac{k}{2}u_x^nv^n-\frac{k}{2}u_x^n-\frac{k}{2}v_x^nu^n$$
$$u^n=\delta_{i-2}^n+26\delta_{i-1}^n+66\delta_i^n+26\delta_{i+1}^n+\delta_{i+2}^n$$
$$v^n=\sigma_{i-2}^n+26\sigma_{i-1}^n+66\sigma_i^n+26\sigma_{i+1}^n+\sigma_{i+2}^n$$
$$u_x^n=\frac{5}{h}(\delta_{i-2}^n+10\delta_{i-1}^n-10\delta_{i+1}^n-\delta_{i+2}^n)$$
$$v_x^n=\frac{5}{h}(\sigma_{i-2}^n+10\sigma_{i-1}^n-10\sigma_{i+1}^n-\sigma_{i+2}^n)$$
$$u_{xx}^n=\frac{20}{h^2}(\delta_{i-2}^n+2\delta_{i-1}^n-6\delta_i^n+2\delta_{i+1}^n+\delta_{i+2}^n)$$
$$v_{xx}^n=\frac{20}{h^2}(\sigma_{i-2}^n+2\sigma_{i-1}^n-6\sigma_i^n+2\sigma_{i+1}^n+\sigma_{i+2}^n)$$
$$u^{n+1}=\delta_{i-2}^{n+1}+26\delta_{i-1}^{n+1}+66\delta_i^{n+1}+26\delta_{i+1}^{n+1}+\delta_{i+2}^{n+1}$$
$$v^{n+1}=\sigma_{i-2}^{n+1}+26\sigma_{i-1}^{n+1}+66\sigma_i^{n+1}+26\sigma_{i+1}^{n+1}+\sigma_{i+2}^{n+1}$$

Equation (8) can be rewritten in the simple form:

$$(1+\frac{5k}{2h}u^n-\frac{20}{6h^2})\delta_{i-2}^{n+1}+\frac{5k}{2h}\sigma_{i-2}^{n+1}+(26+\frac{50k}{2h}u^u-\frac{40}{6h^2})\delta_{i-1}^{n+1}+$$
$$\frac{50k}{2h}\sigma_{i-1}^{n+1}+(66+\frac{120}{6h^2})\delta_i^{n+1}+(26-\frac{50k}{2h}u^n-\frac{40}{6h^2})\delta_{i+1}^{n+1}-$$
$$\frac{50k}{2h}\sigma_{i+1}^{n+1}+(1-\frac{5k}{2h}u^n-\frac{20}{6h^2})\delta_{i+2}^{n+1}-\frac{5k}{2h}\sigma_{i+2}^{n+1}=\eta_i$$
$$\frac{5}{h}(\frac{k}{2}+\frac{k}{2}v^n)\delta_{i-2}^{n+1}+(1+\frac{5k}{2h}u^n-\frac{20}{6h^2})\sigma_{i-2}^{n+1}+\frac{50}{h}(\frac{k}{2}+\frac{k}{2}v^n)\delta_{i-1}^{n+1}$$
$$+(26+\frac{50k}{2h}u^n-\frac{40}{6h^2})\sigma_{i-1}^{n+1}+(66+\frac{120}{6h^2})\sigma_i^{n+1}- \tag{9}$$
$$\frac{50}{h}(\frac{k}{2}+\frac{k}{2}v^n)\delta_{i+1}^{n+1}+(26-\frac{50k}{2h}u^n-\frac{40}{6h^2})\sigma_{i+2}^{n+1}$$
$$-\frac{5}{h}(\frac{k}{2}+\frac{k}{2}v^n)\delta_{i+2}^{n+1}+(1-\frac{5k}{2h}u^n-\frac{20}{6h^2})\sigma_{i+2}^{n+1}=\mu_i\qquad i=0,1,\ldots,N$$

The system in the equation (9) consists of $2N+2$ equation in $2N+10$ unknowns. To get a unique solution to the system, eight additional constraints are required. These are obtained from the boundary conditions (2). Application the boundary conditions enables us to eliminate the parameters $\delta_{-2}^{n+1},\sigma_{-2}^{n+1},\delta_{-1}^{n+1},\sigma_{-1}^{n+1},\delta_{N+1}^{n+1},\sigma_{N+1}^{n+1},\delta_{N+2}^{n+1}$ and σ_{N+2}^{n+1} from the system (9), so the linear system (9) is solved by the Gauss elimination method. To solve the system we apply first the initial conditions to determine:

$(\delta_{-2}^{0},\delta_{-1}^{0},...,\delta_{N+1}^{0},\delta_{N+2}^{0})$ and $(\sigma_{-2}^{0},\sigma_{-1}^{0},...,\sigma_{N+1}^{0},\sigma_{N+2}^{0})$

When t=0,equation (4) takes the formula,

$$U_N^0(x,0)=\sum_{i=-2}^{N+2}\delta_i^0 B_i(x)$$
$$V_N^0(x,0)=\sum_{i=-2}^{N+2}\sigma_i^0 B_i(x)$$

The approximate solution must satisfy the following:

- It must agree with the initial conditions at the knots x_i.
- The derivatives of the approximate initial condition agree with the exact initial conditions at both ends of the range. Eliminating $\delta_{-2}^{0},\sigma_{-2}^{0},\delta_{-1}^{0},\sigma_{-1}^{0},\delta_{N+1}^{0},\sigma_{N+1}^{0},\delta_{N+2}^{0},\delta_{N+2}^{0}$ and σ_{N+2}^{0} with the help of boundary and initial conditions, we obtain the following systems:

$$A\delta^0=B \qquad (10)$$
$$A\sigma^0=D$$

where A is $N+1\times N+1$ square matrix given by:

$$A=\begin{bmatrix} 54 & 60 & 6 & 0 & 0 & 0 & 0 & 0 \\ 25.25 & 67.5 & 26.25 & 1 & 0 & 0 & 0 & 0 \\ 1 & 26 & 66 & 26 & 1 & 0 & 0 & 0 \\ 0 & 1 & 26 & 66 & 26 & 1 & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 1 & 26 & 66 & 26 & 1 \\ 0 & 0 & 0 & 0 & 0 & 26.25 & 67.5 & 25.25 \\ 0 & 0 & 0 & 0 & 0 & 6 & 60 & 54 \end{bmatrix}$$

And

$$\delta^0=\begin{bmatrix}\delta_0^0\\ \delta_1^0\\ \delta_2^0\\ \vdots\\ \delta_{N-1}^0\\ \delta_N^0\end{bmatrix},\quad \sigma^0=\begin{bmatrix}\sigma_0^0\\ \sigma_1^0\\ \sigma_2^0\\ \vdots\\ \sigma_{N-1}^0\\ \sigma_N^0\end{bmatrix}$$

$$B=\begin{bmatrix} u_0+\frac{3h}{5}u_0'+\frac{h^2}{10}u_0'' \\ u_1+\frac{h}{40}u_0'+\frac{h^2}{160}u_0'' \\ u_2 \\ \vdots \\ u_{N-2} \\ u_{N-1}-\frac{h}{40}u_N'+\frac{h^2}{160}u_N'' \\ u_N-\frac{3h}{5}u_N'+\frac{h^2}{10}u_N'' \end{bmatrix}, \quad D=\begin{bmatrix} v_0+\frac{3h}{5}v_0'+\frac{h^2}{10}v_0'' \\ v_1+\frac{h}{40}v_0'+\frac{h^2}{160}v_0'' \\ v_2 \\ \vdots \\ v_{N-2} \\ v_{N-1}-\frac{h}{40}v_N'+\frac{h^2}{160}v_N'' \\ v_N-\frac{3h}{5}v_N'+\frac{h^2}{10}v_N'' \end{bmatrix}$$

The system (10) can be solved by a variant form of Thomas algorithm to get the initial values:

$$\delta_0^0,\delta_1^0,\ldots,\delta_{N-1}^0,\delta_N^0 \text{ and } \sigma_0^0,\sigma_1^0,\ldots,\sigma_{N-1}^0,\sigma_N^0$$

Numerical Results: To illustrate the efficiency of the method, we compute the L_2 and L_∞ error norms;

$$L_2=\left\|u^{exact}-u^{num}\right\|_2=\sqrt{h\sum_{j=0}^{N}\left|u_j^{exact}-u_j^{num}\right|^2}$$

$$L_\infty=\left\|u^{exact}-u^{num}\right\|_\infty=\max_j\left|u_j^{exact}-u_j^{num}\right|$$

To show the well behavior of the numerical procedure.

Single Solitary Wave Motion: The motion of solitary waves is considered in this section. It is well known that system (1) posses analytical solution of the form Chen [18].

$$v(x,t)=-1$$
$$u(x,t)=(1-\frac{g}{6})c+\frac{cg}{2}\operatorname{sec}h^2(\frac{\sqrt{g}}{2}(x+x_0-ct)) \quad (11)$$

where g, x_0 and c are real constants. To compare our results against (11), equation (11) is taken as the initial condition and all computations in the following simulations assume $x_0 = 0$, $g = 6$ and $c=\frac{1}{3}$, for $-20 \le x \le 40$ so that the solitary wave has an amplitude of 1.

Our simulations have been executed up to a time t=20. In Tables 2, 3, 4 and 5, we show the errors L_2, L_∞ at $\Delta t = 0.005$ with various time and space sizes. For the present simulations at t=20 and $\Delta x = 0.04$, $-20 \le x \le 40$, the error norms are $L_2 = 0.040852$, $L_\infty = 0.03953$. In Tables 6, 7, 8 and 9 the L_2, L_∞ error norms is repeated at $\Delta t = 0.001$ with various time and space sizes and it is found that $L_2 = 0.00825$, $L_\infty = 0.0079923$, at $t = 20$, $\Delta x = 0.04$. Solitary wave profiles at time t=0 and t=20 and the error distributions of the Quintic B-spline method and analytic solution at t=20 for $\Delta t = 0.005$, $\Delta t = 0.001$ and $\Delta x = 0.05$ with the range $-20 \le x \le 40$ are shown in Figure 1.

Table 2: The error norms at $t = 5$, $\Delta t = 0.005$, $-20\le x\le 40$

Δx	L_2	L_∞
0.2	0.00404152	0.0043186
0.1	0.00397593	0.0043748
0.066667	0.0039735	0.0043801
0.05	0.00397311	0.0043740
0.04	0.0039730	0.0043806

Table 3: The error norms at $t = 10$, $\Delta t = 0.005$, $-20\le x\le 40$

Δx	L_2	L_∞
0.2	0.011984	0.0125292
0.1	0.01185354	0.012491
0.066667	0.01184844	0.0124897
0.05	0.01184763	0.0124895
0.04	0.01184742	0.0124894

Table 4: The error norms at $t = 15$, $\Delta t = 0.005$, $-20\le x\le 40$

Δx	L_2	L_∞
0.2	0.0243414	0.0238525
0.1	0.024165	0.0241988
0.066667	0.024158	0.0242127
0.05	0.024157	0.0241965
0.04	0.024156	0.024223

Table 5: The error norms at $t = 20$, $\Delta t = 0.005$, $-20\le x\le 40$

Δx	L_2	L_∞
0.2	0.0410763	0.039347
0.1	0.0408625	0.0392898
0.066667	0.0408543	0.0395297
0.05	0.040853	0.0395526
0.04	0.040852	0.0395369

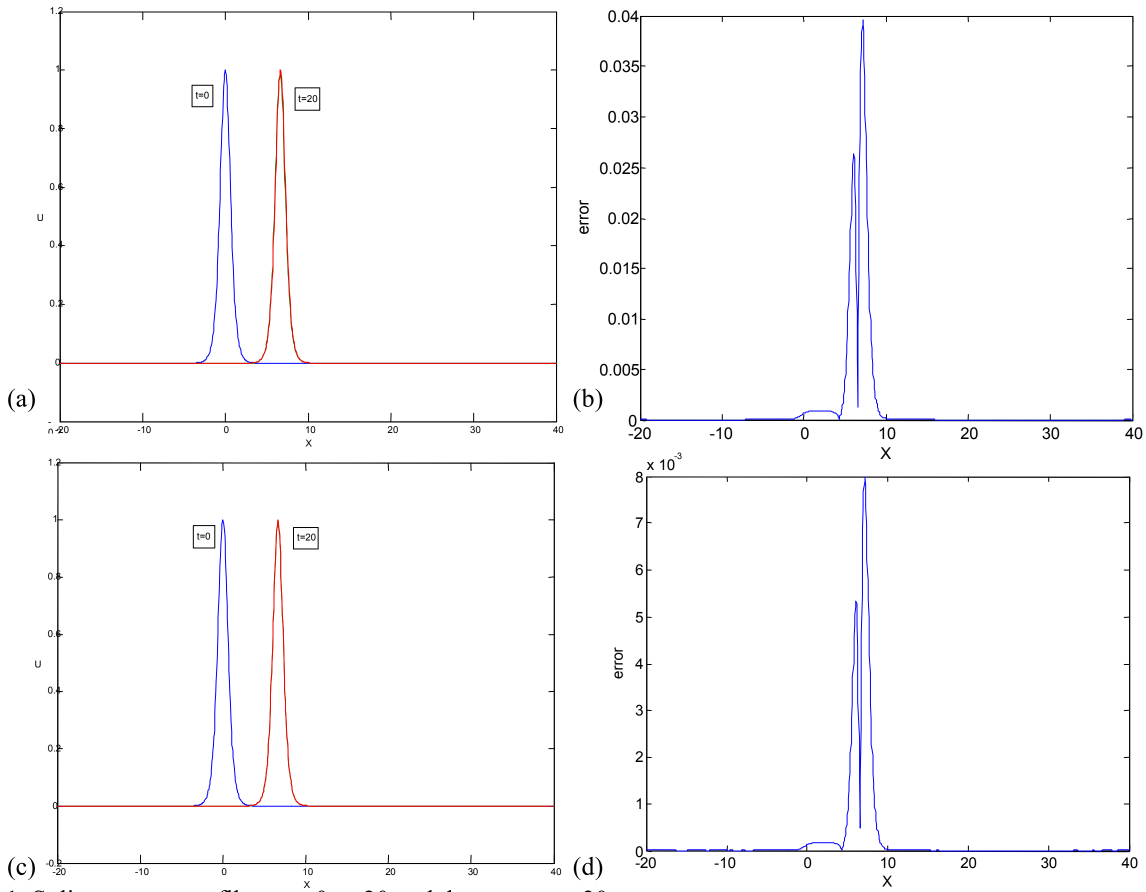

Fig. 1: Solitary wave profiles at t=0, t=20 and the error at t=20

Table 6: The error norms at $t = 5$, $\Delta t = 0.001$, $-20 \leq x \leq 40$

Δx	L_2	L_∞
0.2	0.000885111	0.000897762
0.1	0.000799452	0.000877474
0.066667	0.000796925	0.000878055
0.05	0.000796532	0.000876852
0.04	0.000796427	0.000878022

Table 7: The error norms at $t = 10$, $\Delta t = 0.001$, $-20 \leq x \leq 40$

Δx	L_2	L_∞
0.2	0.00252858	0.0025546
0.1	0.00238702	0.0025111
0.066667	0.00238174	0.0025095
0.05	0.00238091	0.00250932
0.04	0.00238068	0.00250926

Table 8: The error norms at $t = 15$, $\Delta t = 0.001$, $-20 \leq x \leq 40$

Δx	L_2	L_∞
0.2	0.0050641	0.0048636
0.1	0.0048755	0.00488177
0.066667	0.00486836	0.00487511
0.05	0.00486722	0.00487899
0.04	0.00486691	0.00487991

Table 9: The error norms at $t = 20$, $\Delta t = 0.001$, $-20 \leq x \leq 40$

Δx	L_2	L_∞
0.2	0.00849606	0.00806194
0.1	0.008264322	0.00796505
0.066667	0.008255465	0.00797436
0.05	0.008254057	0.00798933
0.04	0.008253663	0.00799238

The traveling waves are graphed at t=0 and t=20 in Figure 1(a). At t=20, both the analytical and numerical solutions are graphed at time t=20 and $\Delta t = 0.005$, the plots of those solutions are indistinguishable. For $\Delta t = 0.005$ the maximum error is about 0.0395369 (Figure 1(b)). On the other hand the observed error at the peak of the wave is 0.0091586636 corresponding to the exact solution at t=20. For $\Delta t = 0.001$, the profiles of the solitary waves are graphed at t=0 and t=20 (Figure 1(c)). Again at t=20, the analytic and numerical solutions are plotted at t=20. Also the solutions are indistinguishable. For $\Delta t = 0.001$, the maximum error is about 0.00799238 (Figure 1(d)). The observed error at the peak of the wave is 0.0015514679 corresponding to exact solution at t=20.

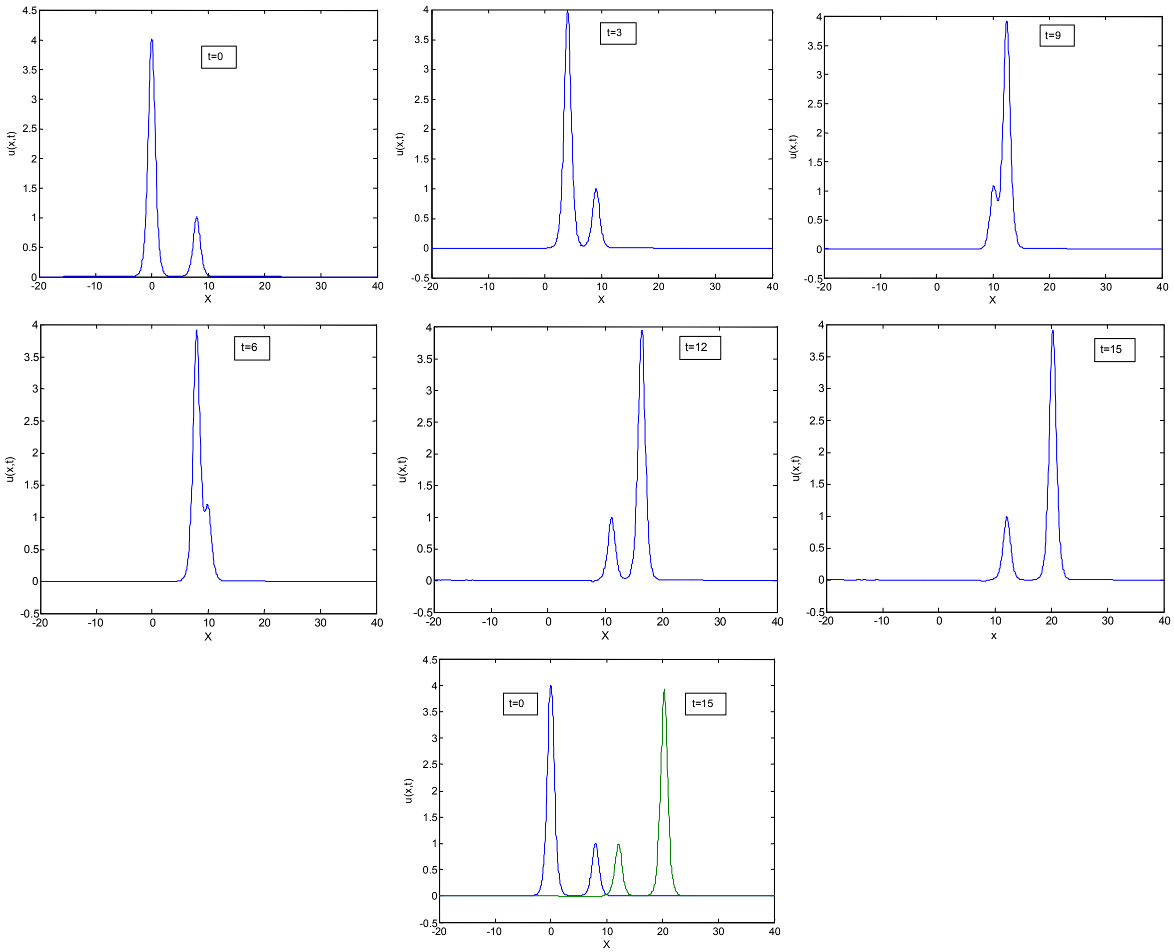

Fig. 2: The motion of two solitary waves at different times

The Interaction of Two Solitary Waves: The interaction of two solitary waves for the coupled BBM-system with the initial condition given by the equations [8] is reported in this section.

$$v(x,t) = -1$$

$$u(x,0) = \sum_{i=1}^{2}\left[(1-\frac{g_i}{6})c_i + \frac{c_i g_i}{2}\sec h^2(\frac{\sqrt{g_i}}{x}(x+x_i)) \right] \quad (12)$$

where g_i, c_i and x_i $i = 1,2$ are real constants. Our system is solved over $-20 \le x \le 40$ with $x_1 = 0$, $x_2 = -10$, $c_1 = 1$ $c_2 = \frac{1}{3}, g_1 = 8, g_2 = 6$, $\Delta t = 0.001$ and $\Delta x = 0.05$. The simulations are executed up to time t=15. In Figure 2, the interaction of two solitary waves is shown, the larger amplitude is 4 at x=0 is on the left of the smaller amplitude is 1 at x=10. After the interaction is finished with complete separation at t=15 the amplitude of the larger wave is 3.9186617884 at x=20.30 whereas the amplitude of the second peak is 0.9912061095 at x=12.15.

CONCLUSIONS

In this paper, a numerical scheme for the nonlinear Coupled BBM-system is presented using Quintic B-spline collocation method. The method has been tested on the motion of single solitary wave and the evolution of two solitary wave interaction. The accuracy of the method was measured using the L_2 and L_∞ error norms. Results reported in this paper revealed that the simulation provides small error. The authors believe that this method is an efficient technique for solving nonlinear partial differential systems.

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