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# Comparative Analysis of Linear and Grafted Polynomial Functions in Forecasting Sorghum Production Trend in Nigeria

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**Abstract:** In agriculture, ex-post and ex-ante forecasts of outputs and market prices, as well as other economic times series data beyond certain periods, cannot be determined with certainty. This paper investigates the performance of linear and grafted polynomial functions in forecasting sorghum production in Nigeria, when the model depends just on the trend. Data were obtained from the Statistical Bulletin of the Central Bank of Nigeria to observe trend in sorghum production from 1970 through 2007. A rough graphical representation indicates that sorghum production did not relate linearly to trend over the entire sample period. A three-time segments function was therefore suggested and estimated after grafting. The resulting mean (grafted) function provided more reliable ex-post forecasts of sorghum production than those yielded from merely fitting a linear function to the data used.

**Key words:** Linear · Grafted Polynomial · Ex-post · Ex-ante · Sorghum

## INTRODUCTION

The forecasting of time series has always been a problem as a result of lack of a functional approach to capture the paucity in available data. For information to be more meaningful and useful for policy making there is need for analysis to refine and summarize the information, hence, the estimation of models to forecast the probable future values of an economic time series is one of the econometric methods of information management [1]. In the past, several studies have utilized models to predict future levels of economic time series. For example, [2] forecasted cotton production in Nigeria using grafted function (Quadratic-Quadratic-Linear) which provided more reliable ex-post forecasts than merely fitting a linear function to the data used. In a related study, [1] forecasted maize production in Nigeria using grafted function (Linear-Quadratic-Linear) which also provided more reliable ex-post forecasts than merely fitting a linear function to the data used. In their study, [3] concluded that there was no universality as to which Spline (grafted) model is appropriate, rather all possible models should be tried and the one that gives the most consistent result when compared to observe data and other factors should be used.

In developing countries like Nigeria, inadequate information on the determinants of an economic time series and the absence of capable statistical routines, have often limited the choice of predictive models [2]. These conditions have necessitated the forecasting of key economic time series (supply, demand, prices, etc) on trend, using variants of the linear model [1].

Some economic time series might exhibit linear relationships over an entire sample period while others may not. This paper therefore, investigates the performance of linear and grafted polynomial functions in forecasting sorghum production in Nigeria, when the model depends just on the trend.

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## MATERIALS AND METHODS

**Data Collection:** The data used were obtained from the Statistical Bulletin of the Central Bank of Nigeria to observe sorghum production trend in Nigeria from 1970 through 2007 (See Appendix C).

**Forecasting Model:** The general form of the linear trend model may be represented as follows:

$$Y = a + bt \qquad (1)$$

Where 'a' and 'b' are structural coefficients, 't' is the trend variable and 'Y' is the level of sorghum production. A rough graphical representation of Y however indicates that the observed time series did not relate linearly to trend 't' over the entire sample period. Then equation (1) has to be improved upon by dividing the available time into segments and using different functional forms for the segments.

The preliminary graphical representation of this time series indicated three time segments and the following trend functions were suggested:

$$Y = a_o + a_1 t, \text{ for } 1970 \leq t \leq 1981 \qquad (2)$$

$$Y = b_o + b_1 t + b_2 t^2 \text{ for } 1981 \leq t \leq 1993 \qquad (3)$$

$$Y = c_o + c_1 t, \text{ for } t > 1993 \qquad (4)$$

In equations (2) - (4), the a's, b's and c's are the structural coefficients, while t and Y are as defined in equation (1).

For purposes of forecasting the time series into the immediate future, it is customary to assume a linear trend relation in the terminal segment [1, 2, 4, 6, 7], that is, equation (4). The aim is to obtain mean function which embodies all the key local trends observed in the time series Y. This mean function must be continuous, linear in the structural coefficients and differentiable at the joints of the pairs of trend functions [1, 2, 4, 6, 7]. In short, it is desired that the following restrictions hold over the domain of the mean function:

$$a_o + a_1 k_1 = b_o + b_1 k_1 + b_2 k_1^2 \qquad (5)$$

$$b_o + b_1 k_2 + b_2 k_2^2 = c_o + c_1 k_2 \qquad (6)$$

$$a_1 = b_1 + 2b_2 k_1 \qquad (7)$$

$$b_1 + 2b_2 k_2 = c_1 \qquad (8)$$

where the k's are the joints of the segmented functions. In this study, $k_1 = 1981$ and $k_2 = 1993$. There are seven (7) structural parameters and four (4) restrictions on the mean function. This means that only 3 parameters can be estimated. In general, which of 7 structural parameters are estimated (or eliminated) depends on the purpose of formulating the mean function. According to [1, 2, 4, 6, 7], where the goal is to forecast, it is important to retain the coefficients in the terminal (usually linear) trend function. Thus, parameters $c_o$, $c_1$ and $b_2$ have been retained for subsequent estimation. This decision resulted to the following solutions (see Appendix A for details):

$$a_o = c_o + b_2 (k_2^2 - k_1^2) \qquad (9)$$

$$a_1 = c_1 - 2b_2 (k_2 - k_1) \qquad (10)$$

$$b_o = c_o + b_2 k_2 \qquad (11)$$

$$b_1 = c_1 + 2b_2 k_2 \qquad (12)$$

The mean function required for the forecasts is obtained by substituting (see Appendix B) for $a_1$, $b_o$ and $b_1$ in equations (2) - (4). The resulting mean function estimated is:

$$Y = c_o X_o + c_1 X_1 + b_2 X_2 \qquad (13)$$

where,

$X_o = 1$ for all t

$X_1 = t$ for all t

$X_2 = [k_2^2 - k_1^2 - 2(k_2 - k_1)t]$ for all $t \leq k_1$

$= (t - k_2)^2$ for $k_1 \leq t \leq k_2$

$= 0$ otherwise.

Equation (13) is now a continuous mean function, given the set of restriction in equations (5) - (8). Adopting the ordinary least squares technique [1, 2, 3, 4, 5, 6, 7 and 8], equations (1) and (13) were each estimated, based on the observed time series on sorghum production in Nigeria from 1970 to 2007. The data for the 1998 – 2007 sub-periods were retained for the ex-post evaluation of the estimated equations.

Table 1: Estimates of the Structural Parameters for Linear and Grafted Functions

| Variables | Linear Function | Grafted Function |
|---|---|---|
| Intercept | -442215.353* | -681667.288* |
| | (-14.946) | (-14.320) |
| $X_1$ | 225.362* | 345.210* |
| | (15.147) | (14.474) |
| $X_2$ | | 11.276* |
| | | (5.651) |
| Adjusted $R^2$ | 0.861 | 0.897 |
| d.f | 37 | 38 |

Notes: Figures in parentheses are the computed t-values
*Significant at 1 percent level of probability

Table 2: Ex-post Forecasts of Sorghum Production in Nigeria, 1998-2007 ('000 tonnes)

| Year | Observed Sorghum Production | Forecasts using Linear Function | Forecasts using Grafted Function |
|---|---|---|---|
| 1998 | 8401 | 8057.5 | 8063.09 |
| 1999 | 8504 | 8282.8 | 8408.3 |
| 2000 | 8824 | 8508.2 | 8753.51 |
| 2001 | 8365.4 | 8733.6 | 9098.72 |
| 2002 | 8712.1 | 8958.9 | 9443.93 |
| 2003 | 9460.8 | 9184.3 | 9789.14 |
| 2004 | 9994.4 | 9409.7 | 10134.35 |
| 2005 | 10593.6 | 9635 | 10479.56 |
| 2006 | 11234.8 | 9860.4 | 10824.77 |
| 2007 | 11769.6 | 10085.7 | 11169.98 |

## RESULTS AND DISCUSSION

The structural coefficients for equations (1) and (13) are presented in Table 1. Equation (1) was estimated for the purpose of evaluating the predictive performance of equation (13). The estimates of the coefficients on $X_1$ and $X_2$ are significant at the 1 percent probability level.

The estimates of the linear and grafted functions in Table 1 were utilized to obtain ex-post forecasts of sorghum production in Nigeria.

Table 2 presents the numerical ex-post forecasts of sorghum production in Nigeria over 1998 – 2007 sub-period, based on the linear and grafted models. The data corresponding to this sub-period was excluded from the regression analysis for evaluating the predictive performance of the grafted function. The ex-post forecasts suggest that the grafted function provides more reliable predictions of the sorghum production during the indicated sub-period.

**Appendix A:** The summary of the derived expressions in coefficients $a_0$, $a_1$, $b_0$ and $b_1$ are already presented in the text as equation (9) – (12). From equations (5) – (8) in the text, it seems easiest to start the derivation with equation (8) and make $b_1$ the subject of the expression as:

$$b_1 = c_1 - 2b_2k_2 \quad \text{(A1)}$$

Equation (A1) can be used to eliminate $b_1$ from equation (7) and then solved to obtain an expression in terms of $a_1$ as:

$$a_1 = c_1 - 2b_2(k_2 - k_1) \quad \text{(A2)}$$

Equation (A2) can be used to eliminate $b_0$ in equation (6). Then, the resulting equation can be expressed in terms of $b_0$ as follows:

$$b_0 = c_0 + b_2k_2^2 \quad \text{(A3)}$$

The expressions obtained above for $b_1$, $a_1$ and $b_0$ can be substituted in equation (5) and rearranged to get an expression in terms of $a_0$ as:

$$a_0 = c_0 + b_2(k_2^2 - k_1^2) \quad \text{(A4)}$$

**Appendix B:** The mean function which was equation (13) in the text was derived simply by substituting for coefficients $a_0$, $a_1$, $b_0$ and $b_1$ as they appear in equations (2) – (4). For equation (2), $t \le k_1$, coefficients $a_0$ and $a_1$ were substituted for, using equations (9) and (10) to get:

$$Y = c_0 + c_1t + b_2[k_2^2 - k_1^2 - 2(k_2 - k_1)t] \quad \text{(B1)}$$

In equation (3), $k_1 \le t \le k_2$, coefficients $b_0$ and $b_1$ were substituted for, using equation (11) and (2) to obtain:

$$Y = c_0 + c_1t + b_2(t - k_2)^2 \quad \text{(B2)}$$

And in equation (4), $t > k_2$, coefficients $c_0$ and $c_1$ were retained for forecasting purposes. The equation remains:

$$Y = c_0 + c_1t \quad \text{(B3)}$$

The mean function equation (equation 13) in the text was formed from the above equations.

## CONCLUSION

The sorghum production trend predicted with the grafted function is closer to the observed trend when compared to that of the linear function because it resulted in smaller differences. This is because the grafted function incorporated the major observed local trends in the forecasting framework. This kind of forecasting technique may be useful for policy making in planning for future crop production based on the available information of the past periods.

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Appendix C: Sorghum Production in Nigeria ('000 tonnes)

| S/No. | Year | Sorghum Output |
|---|---|---|
| 1 | 1970 | 4053 |
| 2 | 1971 | 3794 |
| 3 | 1972 | 2298 |
| 4 | 1973 | 3125 |
| 5 | 1974 | 4738 |
| 6 | 1975 | 2920 |
| 7 | 1976 | 2950 |
| 8 | 1977 | 3286 |
| 9 | 1978 | 2409 |
| 10 | 1979 | 2604 |
| 11 | 1980 | 3346 |
| 12 | 1981 | 3364 |
| 13 | 1982 | 3740 |
| 14 | 1983 | 3292 |
| 15 | 1984 | 4608 |
| 16 | 1985 | 4911 |
| 17 | 1986 | 5455 |
| 18 | 1987 | 5455 |
| 19 | 1988 | 5182 |
| 20 | 1989 | 7265 |
| 21 | 1990 | 4185 |
| 22 | 1991 | 5367 |
| 23 | 1992 | 5909 |
| 24 | 1993 | 6051 |
| 25 | 1994 | 6197 |
| 26 | 1995 | 6997 |
| 27 | 1996 | 7514 |
| 28 | 1997 | 7954 |
| 29 | 1998 | 8401 |
| 30 | 1999 | 8504 |
| 31 | 2000 | 8824 |
| 32 | 2001 | 8365.4 |
| 33 | 2002 | 8712.1 |
| 34 | 2003 | 9460.8 |
| 34 | 2004 | 9994.4 |
| 36 | 2005 | 10593.6 |
| 37 | 2006 | 11234.8 |
| 38 | 2007 | 11769.6 |

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