

Novel Semi-Analytical Approach for Predicting Micro Creep Strain Rates in Reinforced Materials

Vahid Monfared

Department of Mechanical Engineering, Zanjan Branch,
Islamic Azad University, Zanjan, Iran

Abstract: A modern and innovative mathematical model based on transformation function (TF) is introduced for obtaining exact steady state creep strain rate in reinforced materials under axial loading. This new insight is presented for predicting steady state creep behavior of reinforced materials semi-analytically. Results of finite element analysis (FEA) and analytical method are not similar to the prior experimental results in preceding investigations for obtaining steady state creep behavior of fibrous composites. In this research, transformation function is developed to convert analytical and FEA results to the experimental results correctly. This method is approximately exact approach for transforming analytical and FEA results to the experimental results in fibrous composites. Also, fiber behavior is elastic unlike creep behavior of the matrix. This novel idea is a powerful tool for steady state creep analysis in short fiber composites and other relative problems in mechanics. In addition, transformation function is determined semi analytically. Finally, good agreements are found between the results of transformation function method (TFM) and previous experimental results for predicting creep strain rates in short fiber composites (SFC's).

Key words: Reinforced materials • Creep • Semi-analytical method • Analytical • FEA • Short fiber composites

INTRODUCTION

Reinforced materials are among the most powerful candidates as a structural material for many automobile, aerospace, ship and other applications. They do have several interesting characteristics that make them worthy of consideration for other applications. Short fiber composite (SFC) materials have been extensively investigated because they are more economical and impact resistant.

Short fiber composites (SFC), are growing in excellence in aircraft and aerospace industry, automotive and commercial business machine applications. They present the advantage of weight reduction, design flexibility, energy savings and high volume processes for appearance and structural applications. In order to determine the maximum advantages from short fiber composites (SFC's), increasing amounts of researches and development activities have been initiated at many universities and research - laboratory centers.

Many researchers have investigated the steady state creep behavior by analytical and experimental methods. Most of them have studied steady state creep problems by analytical, experimental and finite element method (FEM) without transforming analytical results to experimental results. Of course, some their results were similar to the experimental and finite element methods well. The increasing application of short fiber composites in high loadings and temperatures environments requires a thorough knowledge of their creep characteristics and deformation mechanisms. Recently, extensive investigations have been performed to obtain the creep behavior of short fiber composites in steady state creep and elastic behaviour of composites [1-20].

Some significant and major researches have been carried out in creep of composites and their applications analytically [1-8], experimentally [9-16], numerically [17-20]. For example, shear and radial stress effects on fiber pull-out in interface of fibrous composites has been studied based on shear models [1]. Role of the interfacial properties and some material parameters in the creep

behaviour of discontinuous ductile fiber-reinforced brittle matrix composite systems have been numerically investigated [18]. In addition, finite element methods have been applied for analyzing the continuum aspects of the creep behaviour of unidirectional discontinuous composites loaded parallel and transverse to the fiber axis [19].

Recently, theoretical analyses and interesting methods have been analytically introduced with aim of obtaining comprehensive solutions and algorithms for investigating nonlinear differential and ordinary equations. Furthermore, traveling wave solutions (TWS), direct solutions, integral and iteration methods and such methods have been presented for solving nonlinear equations by various researchers [21-31]. According to the mentioned researches, a simple method should be presented for transforming FEM and analytical results to the experimental results semi-analytically.

The results of the new power transformation function (power TF) for determination of creep strain rate are very exact and wonderful. Creep is more serious in materials that are subjected to high temperatures and loads for long periods and close to melting point.

Also, in the present work, an analytical and mathematical solution is proposed for predicting steady state creep behaviour of short fiber composites based on transformation power function without using some complex theories. For example, this method can be used for analysis of second stage creep in nano short fiber composites, rupture time and mechanical design based on creep.

One of the applications of the strong approach is its ability in analysis of all nano-composites creep, because of high performance in expansion of boundary conditions. Note that, contact surface in nano composites and interface boundary conditions are very extensive.

Therefore, transformation function method (TFM) is unique and perfect method due to high performance in nano-composites creep analysis. In addition, the mentioned method can analyze creep behaviour of short fiber composites in different region of unit cell analytically. Here, an axisymmetric unit cell representing a fiber with its surrounding matrix as two coaxial cylinders is assumed.

For verification of the solution method, the SiC/6061Al composite is selected as a case study and the results will be compared with the previous experimental available results in [10]. New mathematical formulation based on transformation function (TF) is introduced for

determination of exact second stage creep behaviour in short fiber metal matrix composites 6061Al/15%SiC (MMC's) under axial loading.

Analytical steady state creep behaviour of 6061Al/15%SiC (MMC's) composites wasn't similar to the prior experimental behaviour in prior researches. In this research, transformation function is developed for converting previous analytical and FEM results to the experimental results exactly. Transformation function method (TFM) is precise, meticulous and extremely accurate approach for transforming analytical and FEM results to the experimental results in MMC's.

In addition, transformation function is presented semi-analytically. Finally, suitable concurrences are seen between the results of transformation function method (TFM) and prior experimental results for predicting steady state creep behaviour in reinforced materials.

MATERIAL AND METHODS

The cylindrical unit cell depicted in Figure 1 is used to model a short fiber composite. No matrix slipping on the fiber at the interface is an acceptable assumption because of the elastic behavior of the fiber during the creep deformation of the composite.

Therefore, both displacement rate components in the radial and axial directions at the interface are zero. Steady state creep of matrix is schematically shown in Figure 2.

In the mentioned model supposed that a cylindrical fiber with a radius a and a length $2l$ is embedded in a coaxial cylindrical matrix with an outer radius b and a length $2l'$. The volume fraction and aspect ratio of the fiber are defined as f and $s = l/a$ respectively. Also, in this work, $k = (l'/b)(l/a) = l'a/lb$ is considered as a parameter related to the geometry of the unit cell. An applied axial stress $\sigma_{app} = \sigma_0$ is evenly induced on the end faces of the unit cell (at $z = \pm l'$).

Here, creep strain rate relations with respect to applied loads are presented in power, exponential, polynomial and power law functions in the following. These results have been presented in Table 1 and Figures 3, 4, 5 and 6.

Steady state condition of stress is assumed and a full and perfect fiber-matrix interface is considered. Elastic deformations are very small and are neglected as compared to creep deformations. The fibers have elastic behavior during the analysis and the steady state creep behavior of the matrix, which its properties are considered to be constant with temperature, is described by an exponential law as given in Equation (1),

Table 1: Comparison of transformation functions (TF) results with analytical [8], FEM and experimental [10] results

Creep stress (MPa)	Creep strain rates (1/s)						
	Analytical [8]	Experimental [10]	FEM	Power TF	Logarithmic TF	Polynomial TF (n=4)	Exponential TF
50	1.15E-10	7.88261E-11	1.30E-10	8.85E-11	-1.02E-04	-1.66E-09	8.77E-09
60	2.33E-10	7.5163E-10	2.60E-10	8.45E-10	-5.25E-05	7.91E-09	9.64E-09
70	4.74E-10	7.16702E-09	5.20E-10	8.66E-9	-2.91E-06	3.45E-08	1.17E-08
80	9.92E-10	6.83396E-08	1.08E-09	8.49E-8	4.88E-05	1.59E-07	1.77E-08
90	1.95E-09	6.51638E-07	2.26E-09	7.35E-7	9.63E-05	8.90E-07	3.82E-08
100	3.97E-09	6.21356E-06	4.40E-09	7.01E-6	1.46E-04	6.77E-06	1.91E-07
110	8.06E-09	5.92481E-05	9.20E-09	6.69E-5	1.95E-04	5.71E-05	5.05E-06
120	1.64E-08	0.000564948	1.80E-08	0.00063	2.45E-04	5.11E-04	3.88E-03

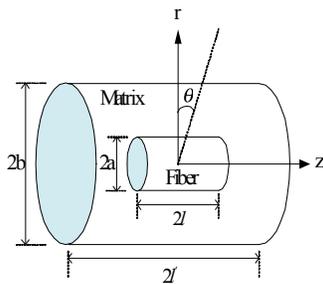


Fig. 1: Presentation of the unit cell schematically.

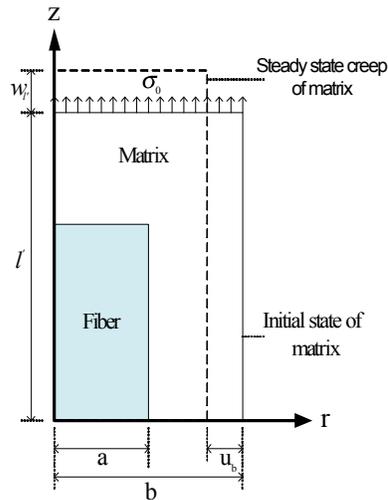


Fig. 2: Schematic model of the unit cell edges in the 2nd stage creep and the elastic states.

$$\dot{\epsilon}_e = A \exp\left(\frac{\sigma_e}{B}\right) \quad (1)$$

Also ϵ_e and σ_e are the equivalent stress and equivalent strain rate of the matrix respectively, will be functions of r and z coordinates. Thus, it turns out to be a very complex nonlinear problem.

For validation and verification of the solution method, the SiC/6061Al composite is selected as a case study and the results will be compared with the previous

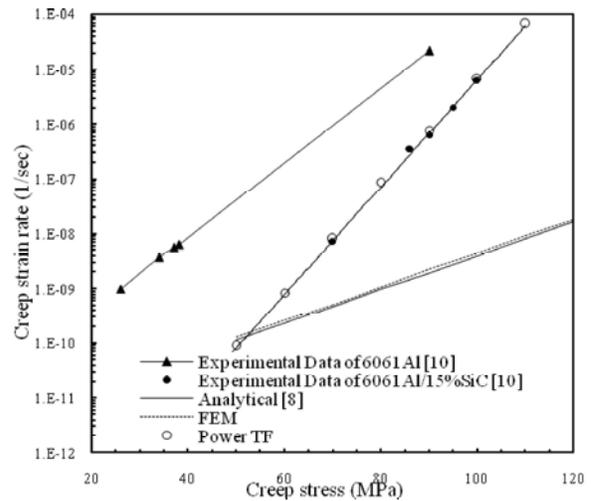


Fig. 3: Effect of power transformation function (Power TF) on analytical [8] and FEM results for a 6061Al/15%SiC in steady state creep at 573 K, (Tensile load).

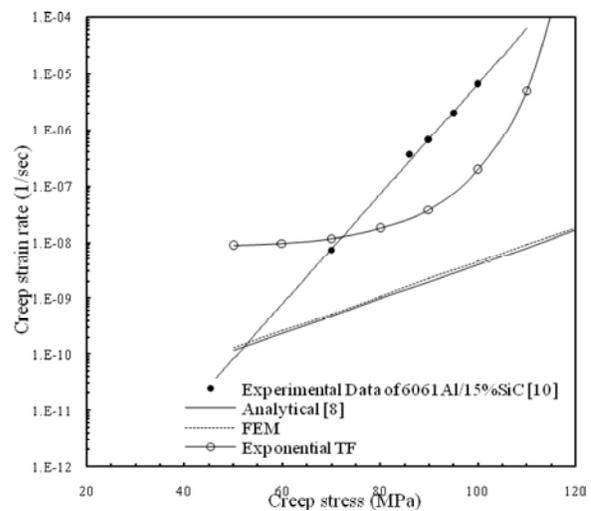


Fig. 4: Effect of exponential transformation function (exponential TF) on analytical [8] and FEM results for a 6061Al/15%SiC steady state creep at 573 K, (Tensile load).

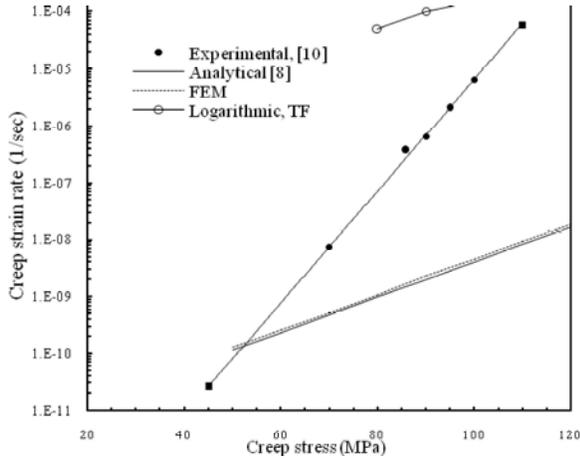


Fig. 5: Effect of exponential transformation function (exponential TF) on analytical [8] and FEM results for a 6061Al/15%SiC steady state creep at 573 K, (Tensile load).

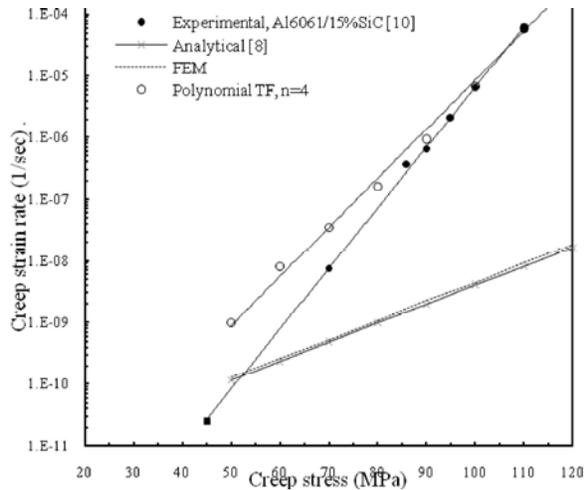


Fig. 6: Effect of polynomial (n=4) transformation function (polynomial TF) on analytical [8] and FEM results for a 6061Al/15%SiC steady state creep at 573 K, (Tensile load).

experimental and analytical results. For the composite used here (SiC/Al6061), the volume fraction (f) of fibers is 15% and the fibers have an aspect ratio (s) of 7.4 and $k = 0.76$ [10]. Also, the steady state creep constants of the matrix material, A and B , in Equation (1) are considered as $A = \exp(-24.7)$ and $B = 6.47$ [10]. In this section, transformation functions (TF) is introduced for determination of exact steady state creep behavior in MMC's (Equations (1)-(7)). This transformation function converts analytical and numerical results on the previous experimental results by power function. This function is given as the following,

$$CSR_{exp} = f(B, A, s, kf, \sigma_0) \quad (2)$$

$$CSR_{exp} \propto B^\theta \times A^\mu \times CSR_{anal(or FEM)}^\varphi \times k \quad (3)$$

$$CSR_{exp} \cong Power TF \quad (4)$$

where unknowns θ, μ, φ are determined by analytical and finite element analysis (FEA) results. These results give, $\theta = \frac{1}{5}, \mu = -2, \varphi = \frac{f\sqrt{sk}}{k^8}, k=1$. In above formulations, "exp" and "anal" indices indicate to the experimental and analytical.

$$CSR_{exp} = \sqrt[5]{B} \times A^{-2} \times CSR_{anal(or FEM)}^{\frac{f\sqrt{sk}}{k^8}} \quad (5)$$

or

$$CSR_{exp} = \frac{2BA^{-2}}{9} \times CSR_{anal(or FEM)}^{\frac{f\sqrt{sk}}{k^8}} \quad (6)$$

In above formulation have $\frac{2B}{9} \cong \sqrt[5]{B}$. Also, A, B are material properties in steady state creep in MMC's. Also, creep strain rate (1/s) with respect to creep stress (MPa) is obtained without using some complex and intricate theories.

Creep strain rate with respect to creep stress has been determined by prior researchers in various and difficult methods. This new approach is very easy, uncomplicated and direct. Thus, according to the results obtained from the Equations (1)-(6), composite creep strain rate $\dot{\epsilon}_e$ is obtained as the following.

$$Ln(\dot{\epsilon}_e) = -34.53 + 0.2255\sigma_{app} \quad (7)$$

where $\sigma_{app} = \sigma_0$ is applied tensile load.

RESULTS AND DISCUSSIONS

Power, polynomial, exponential and logarithmic transformation functions should be analyzed for presentation of transformation function (TF). This analysis shows that power transformation functions are very exact for accurate determination of experimental results. That is, power transformation functions transform the previous analytical and numerical results into the experimental results exactly. Therefore, all the previous analytical and numerical results are transformed to the prior experimental results correctly. The results of these discussions have been shown in figures 3-6 and Table 1.

Effect of exponential transformation function (Exponential TF) on analytical [8] and FEM results for steady state creep in 6061Al/15%SiC at 573 K subjected to tensile loading has been shown in figure 4.

All above figures (figures 3-6) present results of transformation of analytical and numerical results to the experimental results by various transformation functions. Figure 3 shows that the analytical [8] and numerical results are converted to the experimental results correctly.

Table 1 shows that power transformation function is more accurate than the other forms of the transformation functions.

CONCLUSIONS

Semi-analytical and mathematical formulations were developed based on transformation functions (TFs) for determination of steady state creep behavior in short fiber metal matrix composites 6061Al/15% SiC (MMC's) subjected to axial tensile load. Finite element analysis (FEA) and analytical steady state creep behaviors of 6061Al/15%SiC (MMC's) composites were not similar to the prior experimental behaviors in former studies.

Therefore, transformation functions (TFs) were introduced to convert the previous analytical and present FEA results to the experimental results for obtaining creep strain rates correctly.

Transformation function method (TFM) is exact approach for transforming analytical and FEA results to the experimental results in MMC's. Therefore, transformation function was introduced semi-analytically. Eventually, good agreements were established between the results of new transformation function method (TFM) and previous experimental results for predicting creep strain rate behaviors in short fiber composites (SFC's). Power, polynomial (n=4), exponential and logarithmic transformation function results were compared with together to predict composites creep strain rates by transformation function method (TFM). In which, the results were compared with the available experimental data.

In comparison with the other transformation functions, the results of power transformation function and experimental were similar and identical. That is, power transformation function converts the previous analytical [8] and the present FEA results into the experimental results exactly. Therefore, all preceding analytical and FEA results can be transformed to experimental results by power transformation function (Power TF) correctly.

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