Three Level Atom in Bad Cavity

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Abstract: We calculate the gain at the central frequency for a three-level Λ configuration in a bad cavity with a squeezed input. We show that it is possible to obtain the same effects as in the case of the driven three-level Λ system in a free space and damped to a broadband squeezed vacuum. We also show that the coherence induced by the broadband squeezed vacuum can be generated in the case of an input squeezed vacuum. Our results deal with the three-level Λ system and we can easily generalize to the other possible atomic configurations such as V and cascade.

Key words: Broadband squeezed vacuum · bad cavity · atomic coherence · three-level atom · master equation

INTRODUCTION

The two-level atom embedded in a broadband squeezed vacuum has been examined by Gardiner [1] and the fluorescence spectrum has been calculated by Carmichael *et al*. [2, 3]. It has been found that the two quadratures of the atomic polarization decay at two distinct rates that are sensitive to the phase of the squeezed vacuum and the central peak of the fluorescence spectrum could be broadened or narrowed, depending on the relative phase of the driving field and the squeezed vacuum. The squeezing of all modes seen by the atom is difficult to realize experimentally. However, it is experimentally easy to realize a cavity with the squeezed vacuum input on the one side. Parkins and Gardiner [4] have considered a two-level atom inside a microcavity that has squeezed light incident upon the output mirror. They found that the inhibition of atomic decays can still be observed with an input squeezed beam of modest angular dimensions if the phase characteristics of the input beam are suitably matched to the cavity. Rice and Pedrotti [5] have studied a two-level atom in a driven optical cavity coupled to a broadband squeezed vacuum through the output mirror of the cavity. They have calculated the incoherent spectrum of thefluorescent light and have found that the cavity-enhanced spontaneous emission can be suppressed and subnatural linewidth has been obtained, for the correct choice of the phase of the squeezed input relative to the coherent driving field. Courty and Reynaud [6] have also studied a two-level system in a driven bad cavity and they have found that one of the Rabi sidebands can, for the proper detunings and phase of the squeezed light, be suppressed.

Gain from a three-level atom in a bad cavity: A coherent driving field, resonant with both the atomic transition 2↔3 and the cavity, is injected into the cavity through the open sides of the cavity and broadband squeezed vacuum enters the cavity through one mirror (Fig. 1). We assume that the cavity mode frequency and the squeezed vacuum are resonant with the frequency of the atomic transition 3↔2; the pump field is resonant with the frequency of the atomic transition 3↔1 and is treated as classical.

Figure 1 shows the model used by Smyth and Swain [7] (here the atomic system is a three-level Λ system). It describes the interaction of the atom with: the ordinary vacuum (γ1(2) are the consequence of the interaction of the atom with the ordinary vacuum), the coherent field (with the Rabi frequency equal to Ω) and with the cavity mode a. The cavity mode a interacts with the outside modes through the mirror. κ is the damping of the cavity mode a. The

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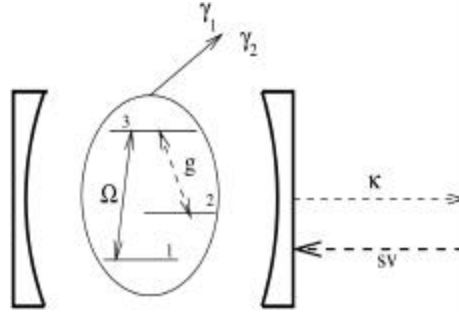
Fig. 1: Three level atom with Λ configuration in a bad cavity. κ is the cavity mode loss, sv is the input squeezed vacuum, $\gamma 1$, $\gamma 2$ are the spontaneous emission from the upper level to the lower ones

cavity mode frequency is chosen to be equal to the atomic transition frequency $\omega 32$, g is the coupling between the cavity mode and the atom. So, the atom interacts with the input field, which is chosen to be the squeezed vacuum sv, through its interaction with the cavity mode a. Following the procedure applied by Smyth and Swain [7] and Rice and Pedrotti [5] we can derive the master equation for our model in the following manner. In a rotating frame, we can generalize the master equation for the two-level system in the cavity [7] to our model, i.e., three-level Λ system and hence, the time evolution of the density operator for the whole system can be written as:

$$
\begin{aligned}
\frac{\partial}{\partial t}\rho = {} & \Omega\left[\sigma_1^- + \sigma_1^+, \rho\right] + g\left[a^+\sigma_2^- + a\sigma_2^+, \rho\right] \\
& - \frac{\gamma_1}{2}\left(\rho\sigma_1^+\sigma_1^- + \sigma_1^+\sigma_1^-\rho - 2\sigma_1^-\rho\sigma_1^+\right) \\
& - \frac{\gamma_2}{2}\left(\rho\sigma_2^+\sigma_2^- + \sigma_2^+\sigma_2^-\rho - 2\sigma_2^-\rho\sigma_2^+\right) \\
& - \kappa(N+1)(\rho a^+a + a^+a\rho - 2a\rho a^+) \\
& - \kappa N(\rho a a^+ + a a^+\rho - 2a^+\rho a) \\
& - \kappa M(\rho a^+a^+ + a^+a^+\rho - 2a^+\rho a^+) \\
& - \kappa M^*(\rho a a + a a\rho - 2a\rho a),
\end{aligned}
$$

The parameters N and M are the usual squeezed vacuum parameters. Here, ρ is the whole system density matrix and we denote the trace over the mode of the cavity by the symbol $<>$.

The evolution of the expectation values of the atomic operators can be found from the provious equation. They are given by

$$
\begin{aligned}
\frac{\partial}{\partial t}\langle\rho_{11}\rangle = {} & i\Omega\langle\rho_{13}\rangle - i\Omega\langle\rho_{31}\rangle + \gamma_1\langle\rho_{33}\rangle, \\
\frac{\partial}{\partial t}\langle\rho_{22}\rangle = {} & ig\langle a\rho_{23}\rangle - ig\langle a^+\rho_{32}\rangle + \gamma_2\langle\rho_{33}\rangle, \\
\frac{\partial}{\partial t}\langle\rho_{33}\rangle = {} & -i\Omega\langle\rho_{13}\rangle - ig\langle a\rho_{23}\rangle + i\Omega\langle\rho_{31}\rangle \\
& + ig\langle a^+\rho_{32}\rangle - (\gamma_1 + \gamma_2)\langle\rho_{33}\rangle, \\
\frac{\partial}{\partial t}\langle\rho_{12}\rangle = {} & ig\langle a\rho_{13}\rangle - i\Omega\langle\rho_{32}\rangle, \\
\frac{\partial}{\partial t}\langle\rho_{13}\rangle = {} & i\Omega\langle\rho_{11}\rangle + ig\langle a^+\rho_{12}\rangle - \gamma_{13}\langle\rho_{13}\rangle \\
& - i\Omega\langle\rho_{33}\rangle, \\
\frac{\partial}{\partial t}\langle\rho_{23}\rangle = {} & i\Omega\langle\rho_{21}\rangle + ig\langle a^+\rho_{22}\rangle \\
& - \gamma_{23}\langle\rho_{23}\rangle - ig\langle a^+\rho_{33}\rangle.
\end{aligned}
$$

From this equations of motion, we see that the mean values of the elements of the density matrix ρ are coupled to the field mode operators a and a+. As Fig. 1 shows, the lower level $|1\rangle$ is not coupled to the cavity mode, so the

equation of motion for the population of the lower level, ρ11, does not include the mode field operator. All the remaining equations of motion for the populations and coherences contain the cavity mode field a. So, the equations of motion contain higher-order expectation values involving both the field and atomic operators. This leads to a hierarchy of coupled equations that must be solved to obtain equations of motion involving atomic operators only. However, in bad cavity limit we adiabatically eliminate the field variables and we find [5] with the properties of the squeezed input

$$
\begin{aligned}
\langle a\,\rho_{23}\rangle &= \frac{i\,g\,N}{\kappa}\langle\rho_{22}\rangle - \frac{-i\,g\,(N+1)}{\kappa}\langle\rho_{33}\rangle, \\
\langle a\,\rho_{13}\rangle &= \frac{i\,g\,N}{\kappa}\langle\rho_{12}\rangle, \\
\langle a^{+}\rho_{12}\rangle &= \frac{i\,g\,(N+1)}{\kappa}\langle\rho_{13}\rangle, \\
\langle a^{+}\rho_{22}\rangle &= \frac{-i\,g\,M^{*}}{\kappa}\langle\rho_{32}\rangle + \frac{i\,g\,(N+1)}{\kappa}\langle\rho_{23}\rangle, \\
\langle a^{+}\rho_{33}\rangle &= \frac{i\,g\,M^{*}}{\kappa}\langle\rho_{32}\rangle + \frac{-i\,g\,N}{\kappa}\langle\rho_{23}\rangle,
\end{aligned}
$$

$$
\begin{aligned}
\langle a^{+}a\rangle &= N, \quad \langle aa^{+}\rangle = N+1, \\
\langle a\,a\rangle &= M, \quad \langle a^{+}a^{+}\rangle = M^{*}.
\end{aligned}
$$

The equations of motion for the density matrix ρ separate from the cavity mode field and then take the form

$$
\begin{aligned}
\frac{\partial}{\partial t}\rho_{11} &= i\Omega\rho_{13} - i\Omega\rho_{31} + \gamma_{1}\rho_{33}, \\
\frac{\partial}{\partial t}\rho_{22} &= -\frac{2g^{2}N}{\kappa}\rho_{22} + \left[\gamma_{2} + \frac{2g^{2}(N+1)}{\kappa}\right]\rho_{33}, \\
\frac{\partial}{\partial t}\rho_{33} &= -i\Omega\rho_{13} + \frac{2g^{2}N}{\kappa}\rho_{22} + i\Omega\rho_{31} \\
&\quad - \left[(\gamma_{1}+\gamma_{2}) + \frac{2g^{2}(N+1)}{\kappa}\right]\rho_{33}, \\
\frac{\partial}{\partial t}\rho_{12} &= -\frac{g^{2}N}{\kappa}\rho_{12} - i\Omega\rho_{32}, \\
\frac{\partial}{\partial t}\rho_{13} &= i\Omega\rho_{11} - \left(\frac{1}{2}(\gamma_{1}+\gamma_{2}) + \frac{g^{2}(N+1)}{\kappa}\right)\rho_{13} \\
&\quad - i\Omega\rho_{33}, \\
\frac{\partial}{\partial t}\rho_{23} &= i\Omega\rho_{21} - \left(\frac{1}{2}(\gamma_{1}+\gamma_{2}) + \frac{g^{2}(2N+1)}{\kappa}\right)\rho_{23} \\
&\quad + \frac{2g^{2}M^{*}}{\kappa}\rho_{32},
\end{aligned}
$$

where we have omitted the brackets denoting the mean values to simplify the notation. The terms in the equations of motion that are proportional to 2g/k can be considered as extra damping rates which correspond to γ2 in the case of a broadband squeezed vacuum. Moreover, the damping rate γ2 which appears in the present equations of motion is the spontaneous emission rate arising from the interaction of the atom with the ordinary vacuum not with the input squeezed vacuum. Furthermore, the squeezed vacuum does not interact with the atom on the pump transition, that is why there are no terms like N γ1. The equations of motion can be written as

$$
\begin{aligned}
\frac{\partial}{\partial t}\rho_{11} &= i\Omega\rho_{13} - i\Omega\rho_{31} + \gamma_{1}\rho_{33}, \\
\frac{\partial}{\partial t}\rho_{22} &= -\tilde{\gamma}_{2}N\rho_{22} + [\gamma_{2} + \tilde{\gamma}_{2}(N+1)]\rho_{33}, \\
\frac{\partial}{\partial t}\rho_{33} &= -i\Omega\rho_{13} + \tilde{\gamma}_{2}N\rho_{22} + i\Omega\rho_{31} \\
&\quad - [(\gamma_{1}+\gamma_{2}) + \tilde{\gamma}_{2}(N+1)]\rho_{33},
\end{aligned}
$$

$$\frac{\partial}{\partial t}\rho_{12} = -\gamma_{12}\rho_{12} - i\Omega\rho_{32},$$

$$\frac{\partial}{\partial t}\rho_{13} = i\Omega\rho_{11} - \gamma_{13}\rho_{13} - i\Omega\rho_{33},$$

$$\frac{\partial}{\partial t}\rho_{23} = i\Omega\rho_{21} - \gamma_{23}\rho_{23} + \tilde{\gamma}_2 M^*\rho_{32},$$

where

$$\tilde{\gamma}_2 = \frac{2g^2}{\kappa}$$

$$\gamma_{12} = \frac{\tilde{\gamma}_2}{2}N,$$

$$\gamma_{13} = \frac{1}{2}(\gamma_1 + \gamma_2) + \frac{\tilde{\gamma}_2}{2}(N+1),$$

$$\gamma_{23} = \frac{1}{2}(\gamma_1 + \gamma_2) + \tilde{\gamma}(N + \frac{1}{2}),$$

Steady state solutions and spectra: Once the master equation has been derived, the absorption spectrum and the fluorescence spectrum can be calculated. Before calculating the spectra, we calculate the steady state solutions. The steady state solutions for the populations of the lasing levels are

$$\rho_{22} = 2\Omega^2 \frac{\gamma_2 + \tilde{\gamma}_2(N+1)}{2(\gamma_2 + \tilde{\gamma}_2(3N+1))\Omega^2 + \gamma_1\tilde{\gamma}_2\gamma_{13}N},$$

$$\rho_{33} = 2\Omega^2 \frac{\tilde{\gamma}_2 N}{2(\gamma_2 + \tilde{\gamma}_2(3N+1))\Omega^2 + \gamma_1\tilde{\gamma}_2\gamma_{13}N},$$

$$\rho_{31} = \Omega \frac{-i\gamma_1\tilde{\gamma}_2 N}{2(\gamma_2 + \tilde{\gamma}_2(3N+1))\Omega^2 + \gamma_1\tilde{\gamma}_2\gamma_{13}N},$$

$$\rho_{11} = \frac{N\tilde{\gamma}_2(\gamma_1\gamma_{13} + 2\Omega^2)}{2(\gamma_2 + \tilde{\gamma}_2(3N+1))\Omega^2 + \gamma_1\tilde{\gamma}_2\gamma_{13}N}.$$

One can see that the population of the level $|2>$ is always bigger than the population of the upper level $|3>$, there is no population inversion between the lasing levels. We can also derive the steady state solution for the dressed states. One can found levels. Using the steady state solutions we can calculate the absorption spectrum. It is given by

$$F(\delta_p) = \text{Re}\left\{ \frac{\Gamma_z\left[(z + \gamma_{12})\rho_{33} + i\Omega\rho_{31}\right]}{\Gamma_z^2 - \left[(z + \gamma_{12})|M|\tilde{\gamma}_2\right]^2}\bigg|_{z=i\delta_p}\right\},$$

One can see that the absorption spectrum takes similar form as for the broadband squeezed vacuum. We can find the fluorescence spectrum in the same manner. It takes the form

$$A(\delta_p) = \text{Re}\left\{ \frac{\Gamma_z\left[(z + \gamma_{12})(\rho_{22} - \rho_{33}) - i\Omega\rho_{31}\right]}{\Gamma_z^2 - \left[(z + \gamma_{12})|M|\tilde{\gamma}_2\right]^2}\bigg|_{z=i\delta_p}\right\},$$

The negative values of Eq. (18) give amplification, while its positive values give absorption. The condition for amplification at central line can be found by substituting dp = 0. We have found that A(0) < 0 leads to γ1 > γ2 + γ2.

We can see from Eq. (21) that when κ » γ1(2) we can neglect γ2 and get the condition for amplification without population inversion for the system that is driven by one resonant coherent field and pumped incoherently on the other transition. For the bad cavity the damping rate γ1 must be bigger than γ2 plus γ2, which is an extra damping rate resulting from the effect of the cavity on the atom.

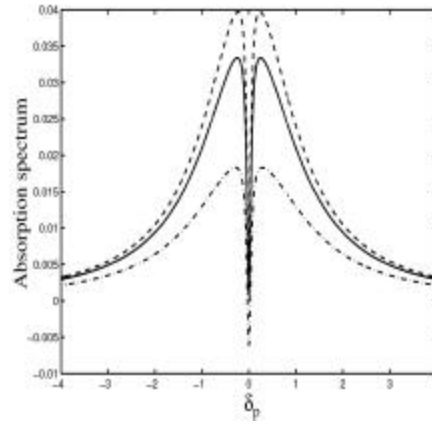Fig. 2:   Three absorption spectrum on the transition 2↔3 versus the probe detuning for Λ-system in a bad cavity with a squeezed input which has a frequency equal to the probe transition and pumped on the transition 1↔3 by a coherent field
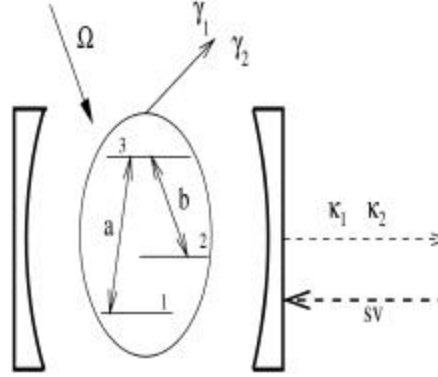

Fig. 3:   Three level atom in a two-mode cavity pumped by a coherent field Ω, damped into ordinary vacuum ($\gamma 1$, $\gamma 2$) and interacting with the two modes a and b. The two cavity modes (a, b) interact with the input squeezed vacuum reservoir through a loosely mirror

For the high Rabi frequencies the absorption spectrum shows two peaks at the Rabi frequencies. These two symmetrically placed peaks show absorption because there is no population inversion between the dressed states involved.

For low Rabi frequencies, we can expect a dip at the central line, which shows amplification and there is transparency if we have $\gamma 1 = \gamma 2 + \gamma 2, A(0) = 0$. If we can choose our atomic system in such a way that the relation for the transparency is verified $\gamma 1 = \gamma 2 + \gamma 2$, whenever the Rabi frequency is small we always get a dip at the central line and if the field is a little bit shifted from the resonance it exhibits strong absorption (Fig. 2). For the broadband squeezed vacuum the condition for amplification without population inversion is that the Rabi frequency must exceed a threshold value Ωth. However, in our present case we only need the condition $\gamma 1 > \gamma 2 + \gamma 2$ between the damping constants to be satisfied. The condition for amplification without population inversion depends only on the damping constants if the mean number of photons is zero. It becomes $\gamma 1 > \gamma 2$ and this is a similar condition to that in a bad cavity, the amplification without population inversion depends only on the difference between the damping rates.

$$\bar{\gamma}_2 = \frac{2\,g^2}{\kappa}$$

$$\gamma_{12} = \frac{\bar{\gamma}_2}{2} N,$$

$$\gamma_{13} = \frac{1}{2}(\gamma_1 + \gamma_2) + \frac{\bar{\gamma}_2}{2}(N+1),$$

$$\gamma_{23} = \frac{1}{2}(\gamma_1 + \gamma_2) + \bar{\gamma}(N+\frac{1}{2}),$$

Atomic coherences induced in bad cavity: Let us consider a model in which a nondegenerate three-level atom in the Λ configuration placed in a two-mode cavity is coupled to a two mode squeezed vacuum through the output mirror and driven by a coherent field. It is well known that if the atom possesses a non-zero coherence transfer rate and is driven by one field and damped into a broadband squeezed vacuum, all the coherences can be induced. Since it is difficult to realize the model of perfect broadband squeezed vacuum in which the atom can see all the modes squeezed, we assume that the atom is located in a two-mode cavity. These two modes are resonant with the atomic frequencies and the squeezed vacuum frequency is taken to be equal to the half of the sum of the two allowed atomic transition frequencies (Fig. 3).

The equation of motion can be derived in the same way as in the previous section. It takes the form

$$
\frac{\partial}{\partial t}\rho_{11} = -N\bar{\gamma}_1\rho_{11} + i\Omega_1\rho_{13} - i\Omega_1\rho_{31}
$$
$$
+[(N+1)\bar{\gamma}_1 + \gamma_1]\rho_{33},
$$
$$
\frac{\partial}{\partial t}\rho_{22} = -N\gamma_2\rho_{22} + (N+1)\gamma_2\rho_{33},
$$
$$
\frac{\partial}{\partial t}\rho_{12} = -\gamma_{12}\rho_{12} - i\Omega_1\rho_{32},
$$
$$
\frac{\partial}{\partial t}\rho_{13} = i\Omega_1\rho_{11} - \gamma_{13}\rho_{13} + M^*\bar{\gamma}\rho_{32}
$$
$$
-i\Omega_1\rho_{33},
$$
$$
\frac{\partial}{\partial t}\rho_{23} = i\Omega_1\rho_{21} - \gamma_{23}\rho_{23} + M^*\bar{\gamma}\rho_{31},
$$

$$
\rho_{33} = 1 - \rho_{22} - \rho_{33},
$$
$$
\rho_{ji} = \rho_{ij}^*,
$$

$$
\bar{\gamma}_1 = 2\frac{g_1^2}{\kappa_1},
$$
$$
\bar{\gamma}_2 = 2\frac{g_2^2}{\kappa_2},
$$
$$
\bar{\gamma} = g_1 g_2 \left[\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right].
$$

The steady state solution for the coherence ρ_{23} takes the same form as for broadband squeezed vacuum

$$
\rho_{23} = \frac{-i\Omega M^*\bar{\gamma}\gamma_{12}}{\bar{\gamma}_{13}(\gamma_{12}\gamma_{23} + \Omega^2)}(\rho_{11} - \rho_{33}),
$$
$$
\bar{\gamma}_{13} = \gamma_{13} - \frac{\gamma_{12}|M|^2\bar{\gamma}^2}{\gamma_{12}\gamma_{23} + \Omega^2}.
$$
$$
\rho_{11} - \rho_{33} = \frac{N\bar{\gamma}_2(\gamma_1 + \bar{\gamma}_1)\bar{\gamma}_{13}}{N\bar{\gamma}_{13}[\bar{\gamma}_1\gamma_0 + (\bar{\gamma}_1 + \gamma_1)\bar{\gamma}_2] + 2\Omega^2\gamma_0},
$$

So, we can generate the coherence ρ_{23} without pumping the transition $2\leftrightarrow3$ by any coherent field. This feature is present in a broadband squeezed vacuum as well as in a cavity with the input squeezed vacuum.

CONCLUSION

In this paper we have studied the interaction of a three-level Λ system with one coherent field. The atom is in a bad cavity with a squeezed vacuum input. In the first Section we have chosen the squeezed vacuum frequency to be equal to the probe atomic transition frequency. We have derived the master equation and then calculated the absorption spectrum and fluorescence spectrum. We have shown that the equations of motion are similar to those

when the atom is damped into a broadband squeezed vacuum and thus, the squeezed vacuum enhances the amplification of the probe field without affecting the steady state solutions. In the second Section we have used a two-mode squeezed vacuum with the property that the squeezed vacuum couples the modes a and b which act on the allowed atomic transitions. We have shown that it is possible to generate steady state atomic coherences using the two-mode squeezed vacuum input. The coherence transfer rate which is crucial for generating the coherences is now in the case of the input squeezed vacuum, given in the text. So, the property of generating atomic coherences in the case of a broadband squeezed vacuum can also be observed for the atom placed in the cavity with the input squeezed vacuum.

ACKNOWLEDGMENTS

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