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Application of Compressible Vorticity Confinement Methods for Solving Different Vortex Dominant Flows

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Abstract: In this paper four different vorticity confinement methods were used to solve three vortex dominant problems. These four vorticity confinement methods were scalar, matrix, lambda and adaptive methods. Three test cases in two-dimensional inviscid flow were used in order to assess the capability of these vorticity confinement methods. These test cases were two dimensional vortex in a uniform flow, supersonic shear layer and shock-vortex interaction, respectively. The results showed that the lambda method is effective enough. Also this formulation (lambda) is faster than the matrix vorticity confinement method. The adaptive method has less accuracy in contrast to the other vorticity confinement methods, but its real worth is eliminating empirical constant coefficients. So this method (adaptive method) is faster than the other vorticity confinement methods and thus it is an effective method.

Key words: Adaptive · Compressible Flows · Vortex dominant · Vorticity Confinement

INTRODUCTION

Many flows of interest are characterized by large regions of concentrated vortical structures that persist and can convect over long distances, such as flows around aircrafts, ships, automobiles, bridges and buildings. Conventional CFD methods tend to dissipate vortical structures, degrading the overall accuracy of the computed flows. This dissipation can be reduced through using fine grids, but at the expense of greatly increased computational demands.

Therefore, vorticity confinement method has been proposed to reduce the diffusive property of the incompressible and compressible vortical flow simulations by Steinhoff and co-workers [1, 2]. In this method, the source term added to the Navier-Stokes equations works as it convects the discretization error back into the vortex center and thus confines the vortex. This has the effect of adding a velocity correction that convects vorticity in the opposite direction of numerical diffusion [3].

However, in spite of the success of the method with regard to its goals, the confinement term is proportional to an empirical parameter (E_c) that is constant over the entire flow field and has a dimension of velocity. Using a constant value over the entire flow field is questionable.

In order to make confinement parameter dependent on flow and computational parameters, several studies attempted different ideas: Fedkiw *et al*. [4] introduced the mesh size h to guarantee the computational consistency with mesh refinement. Lohner and Yang [5] devised a formulation where a proper length scale h is computed by considering the gradient of vorticity magnitude and the confinement term is proportional to h^2. Robinson [6] suggested a formulation where the confinement term is proportional to helicity. Butsuntorn and Jameson [7] devised a formulation where the confinement term is proportional to the logarithm of cell-volume ratio and helicity magnitude in their rotor-blade simulation using the time-spectral method.

In order to make confinement parameter dependent on flow and computational parameters, Hu *et al*. [2] suggested a formulation that the confinement term was proportional to the flux Jacobian matrix. This method was called as matrix vorticity confinement method. But calculation of the flux Jacobian matrix is complicated and needs a lot of time in contrast to the other vorticity confinement methods. So Bagheri Esfe and Malek Jafarian [8] suggested a new confinement parameter (E_{cL}) which was proportional to spectral radii of the flux Jacobian matrix. This method was named as lambda formulation.

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The above mentioned studies introduce an improved dependency of confinement terms on flow and mesh properties, but they don't completely eliminate the proportionality constant (E_c).

Hahn and Iaccarino [9] suggested a procedure for an improved prediction of vortical fields in the presence of numerical dissipation that was the adaptive vorticity confinement. In this method, the strength of the confinement term is computed without any need for an arbitrary constant. That formulation was suggested for incompressible flows. Then Bagheri Esfe and Malek Jafarian [8] developed the Hahn and Iaccarino's formulation to compressible flow fields. This method was named as adaptive method.

In this paper four different vorticity confinement methods (scalar, matrix, lambda and adaptive methods) are used to solve three different problems. These test cases are two dimensional vortex in a uniform flow, supersonic shear layer and shock-vortex interaction. These test cases in two-dimensional inviscid flow are used in order to assess the capability of different vorticity confinement methods.

Governing Equations For Different Vorticity Confinement Methods: Considering the Euler equations in Cartesian coordinate:

$$\frac{\partial Q}{\partial t}+\frac{\partial f}{\partial x}+\frac{\partial g}{\partial y}=S \tag{1}$$

Where

$$Q=\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho e_\circ\end{pmatrix},\quad f=\begin{pmatrix}\rho u\\ \rho u^2+P\\ \rho uv\\ \rho uh_\circ\end{pmatrix},\quad g=\begin{pmatrix}\rho v\\ \rho vu\\ \rho v^2+P\\ \rho vh_\circ\end{pmatrix} \tag{2}$$

$$h_\circ=e_\circ+\frac{P}{\rho},\quad P=(\gamma-1)\times\rho\times\left\{e_\circ-\frac{u^2+v^2}{2}\right\} \tag{3}$$

$$S=\begin{pmatrix}0\\ \rho f_b.\hat{i}\\ \rho f_b.\hat{j}\\ \rho f_b.V\end{pmatrix} \tag{4}$$

Q is the vector of conserved variables, f and g are the flux vectors and S is vorticity confinement vector. The independent variables are time (t) and Cartesian coordinates (x, y). ρ, u, v, e and P denote non-dimensional density, Cartesian velocity components, energy and pressure, respectively. f_b is a body force per unit mass which tries to balance the numerical diffusion inherently related to numerical discretization and conserves momentum in vortical regions:

$$f_b=-E_c\hat{n}_c\times\omega \tag{5}$$

where

$$\hat{n}_c=-\frac{\nabla f}{|\nabla f|},\ \varphi=|\omega|=\sqrt{\omega_x{}^2+\omega_y{}^2}\Rightarrow\hat{n}_c=\varphi_{xs}\hat{i}+\varphi_{ys}\hat{j} \tag{6}$$

$$f_{xs}=\frac{f_x}{\sqrt{f_x{}^2+f_y{}^2}},\ f_{ys}=\frac{f_y}{\sqrt{f_x{}^2+f_y{}^2}} \tag{7}$$

Finally, the vorticity confinement term in a compressible flow can be expressed as follows:

$$S=\begin{pmatrix}0\\ \rho f_{bx}\\ \rho f_{by}\\ \rho(uf_{bx}+vf_{by})\end{pmatrix}=\begin{pmatrix}0\\ -\rho E_c\omega_z\varphi_{ys}\\ \rho E_c\omega_z\varphi_{xs}\\ -\rho E_c\omega_z(u\varphi_{ys}-v\varphi_{xs})\end{pmatrix} \tag{8}$$

where E_c is the scalar confinement parameter with the dimension of velocity. Also it is an empirical constant coefficient. ω_z is the z-component of vorticity vector (ω).

Assuming a constant coefficient for the scalar confinement parameter (E_c) is questionable. Because it has the dimension of velocity and should depend on the local conditions of flow field.

For solving this problem, the matrix confinement parameter was derived from the flux Jacobian matrix by Hu *et al.*[2]. The local conditions of flow field exist in the formulation of matrix vorticity confinement by flux Jacobian matrices. This is the preference of matrix vorticity confinement in contrast to the scalar vorticity confinement.

The matrix vorticity confinement term is defined as:

$$S=S_x+S_y \tag{9}$$

$$S_x=|A|\begin{pmatrix}0\\ E_{cc}\rho(\hat{n}_c\times\omega).\hat{i}\\ 0\\ E_{cc}\rho(\hat{n}_c\times\omega).u\hat{i}\end{pmatrix},S_y=|B|\begin{pmatrix}0\\ 0\\ E_{cc}\rho(\hat{n}_c\times\omega).\hat{j}\\ E_{cc}\rho(\hat{n}_c\times\omega).v\hat{j}\end{pmatrix} \tag{10}$$

where E_{cc} is the matrix confinement parameter with the dimension of velocity. For more details of matrix $|A|$ and matrix $|B|$ see [2].

Althogh the local conditions of flow field exist in the formulation of matrix vorticity confinement by flux Jacobian matrices, but the calculation of flux Jacobian matrices needs a lot of time and great memory storage.

For solving this problem, a new formulation was suggested for vorticity confinement parameter by Bagheri Esfe and Malek Jafarian [8]. The flux Jacobian matrix was replaced with the spectral radii of flux Jacobian matrix in this new formulation:

$$E_c = |\lambda| \times E_{cL} \tag{11}$$

$|\lambda|$ is the spectral radii of flux Jacobian matrix and has the dimension of $\frac{m^3}{s}$. This new vorticity confinement method was named as lambda method.

The accuracy of the lambda method is almost equal to the matrix vorticity confinement method. But the computational time for this new method is almost half of the matrix method. So the new formulation for vorticity confinement is an effective method.

The conventional vorticity confinement methods have a empirical constant parameter (confinement parameter such as E_c, E_{cc}, E_{cL}) that should be determined for every problem by trial and error. Different constant values should be examined for every problem and the best value should be selected for that problem. Selecting the best value for these constant coefficients needs a lot of time. In order to gain confidence in the applicability of vorticity confinement, it would be ideal to completely eliminate such constant parameters.

Hahn and Iaccarino [9] suggested a procedure for an improved prediction of vortical fields in the presence of numerical dissipation that was the adaptive vorticity confinement. That formulation was suggested for incompressible flows. Then Bagheri Esfe and Malek Jafarian [8] developed the Hahn and Iaccarino's formulation to compressible flow fields. This method was named as adaptive method. This new vorticity confinement method doesn't need any constant coefficient. Thus it is much faster in contrast to the other vorticity confinement methods and it's an effective method.

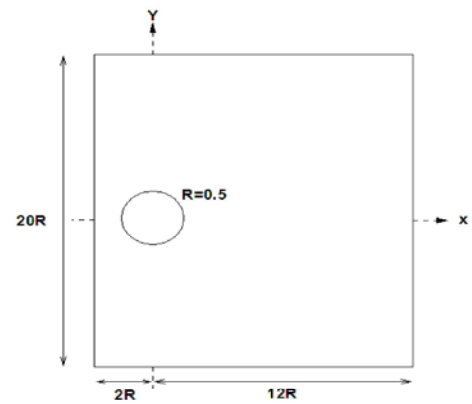

Fig. 1: The computational domain for the moving vortex in a uniform flow

RESULTS AND DISCUSSION

In this part, different vorticity confinement methods are compared for three different problems: Two dimensional vortex in a uniform flow, supersonic shear layer and shock-vortex interaction.

Two Dimensional Vortex In a Uniform Flow: A special vortex at the point (0,0) is considered in Figure 1. The Mach number of the uniform flow is 0.5. More details of this problem are in [2]. A 350×250 grid is considered. The grid is uniform along the x-axis and is distributed according to an exponential function along the y-axis, so that the grid becomes finer as the y coordinate limits to the zero.

The analytic solution for a compressible vortex moving with a uniform free stream is discussed in [2]. The analytical solution shows that the quantity ω/ρ of the vortex will be conserved as it convects in a compressible, inviscid fluid. Then different vorticity confinement methods are compared. The results are presented in Figure 2.

In Figure 2, the quantity of ω/ρ is shown for different vorticity confinement methods when the center of the vortex reaches the position x=2.98. For each method, different values of vorticity confinement parameters have been examined and the best values have been selected. In Figure 2, E_c, E_{cc} and E_{cL} show the scalar, matrix and lambda vorticity confinement parameters, respectively. Also this figure depicts adaptive vorticity confinement method that doesn't need any constant coefficient. One can see that the vorticity confinement terms effectively balance the numerical dissipations. The best

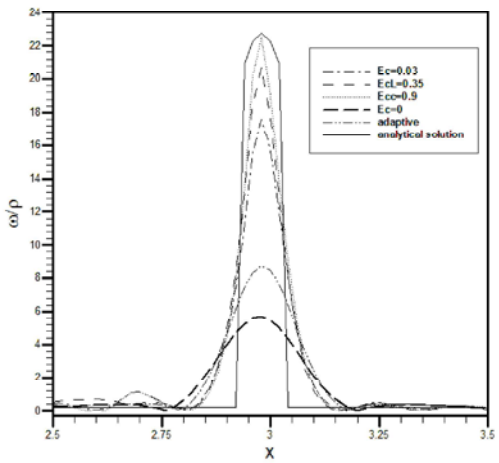

Fig. 2: x = 2.98, ω/ρ Distribution for the moving vortex when the center of the vortex is at the position $x=2.98$, Comparison between different vorticity confinement methods

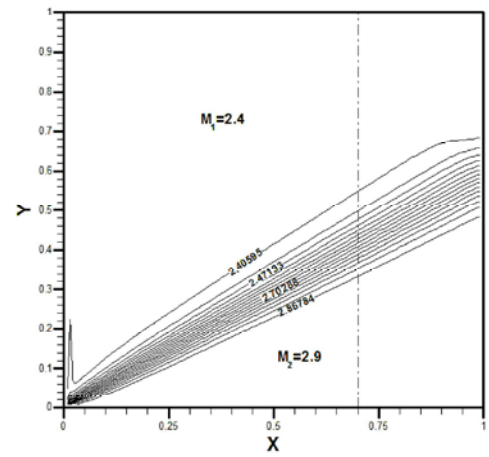

Fig. 3: Mach contours of Inviscid supersonic shear layer, no vorticity confinement, after 1000 time steps

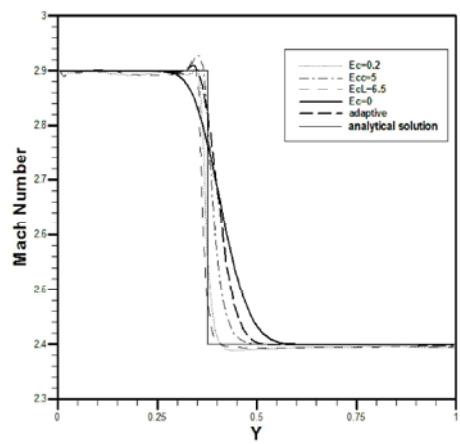

Fig. 4: Mach number distribution along a vertical cut at the position x=0.7 in the supersonic shear layer, after 1000 time steps

result is related to the matrix vorticity confinement method with E_{cc} = 0.9, because its result is closer to the analytical solution in contrast to the other methods. But as pointed earlier, this method needs a lot of calculations.

Figure 2 shows that the lambda confinement parameter (E_{cL} = 0.35) has appropriate accuracy. Also the computational time of this method is much less than the computational time of the matrix method. Thus it is concluded that the lambda confinement parameter (E_{cL}) is effective.

Also Figure 2 shows that the adaptive method has less accuracy in contrast to the other vorticity confinement methods. But as pointed earlier, it doesn't need any constant coefficient. Other vorticity confinement methods have this empirical constant coefficient and it takes a lot of time to determine the best value of this coefficient for different methods. Thus the adaptive method is much faster in contrast to the other vorticity confinement methods.

Supersonic Shear Layer Problem: An inviscid supersonic shear layer flow is formed by two parallel streams, one at Mach number M_1 = 2.4 and the other at M_2 = 2.9. Two streams at angle of 30° relative to the horizontal grid line are considered to show that the solution is independent from the direction of the grid. The computational grid is a 100×100 uniform Cartesian grid. Figure 3 shows Mach contours of Inviscid supersonic shear layer, without vorticity confinement.

Figure 4 presents the Mach numbers along a cut line that parallels to the vertical axis at (x=0.7). Without confinement the solution is significantly degraded. This is due to numerical dissipations. Using different vorticity confinement methods will sharpen the discontinues. Different values of vorticity confinement parameters have been examined and the best values have been selected for every vorticity confinement parameter. Figure 4 shows that the best result is related to the case E_{cL} = 6.5. Because the local properties of flow field are considered in this new formulation of vorticity confinement, this method has the best answer for this problem. Also the computational time of this method is much less than the computational time of the matrix method.

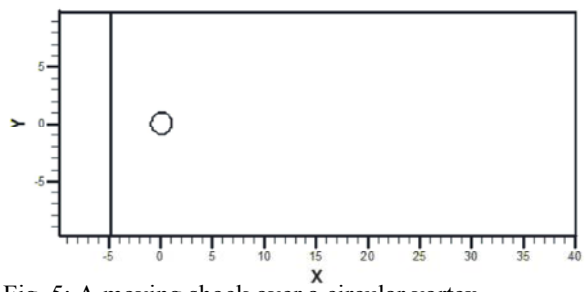

Fig. 5: A moving shock over a circular vortex

Also Figure 4 shows that the adaptive method has less accuracy in contrast to the other vorticity confinement methods, but as pointed earlier, its real worth is eliminating the empirical constant coefficients from vorticity confinement parameters. Thus this method is much faster in contrast to the other vorticity confinement methods.

Shock Vortex Interaction: We want to compute the problem of a moving shock over a circular vortex shown in Figure 5. The Mach number of the shock wave is set to 1.5.

As we know, the inner region of the vortex has less density relative to the outer region. Thus the relative density is introduced as follows:

Table 1: The time related to the different steps of shock-vortex interaction

Different steps	(μs) Time
b-1,c-1	50
b-2,c-2	70
b-3,c-3	90
b-4,c-4	110
b-5,c-5	157
b-6,c-6	203

$$D = \frac{\rho_L}{\rho_h} < 1 \tag{12}$$

where ρ_L is the density of region inside the vortex and ρ_h is the density of region outside the vortex.

The computational grid is $50R_o \times 20R_o$. The grid is uniform along the x-axis and is distributed according to an exponential function along the y-axis, so that the grid becomes finer as the y coordinate limits to zero.

Figure 6 shows the density contours related to different steps of shock-vortex interaction. The density contours shown here are for the grid (1000 × 200) using scalar vorticity confinement ($E_c = 0.02$) and scalar artificial dissipation scheme. They are compared to the adaptive unstructured algorithm results of Povitsky *et al*. [10] in Figure 6. This figure shows the results of different time steps. These time steps are shown in Table 1. The results

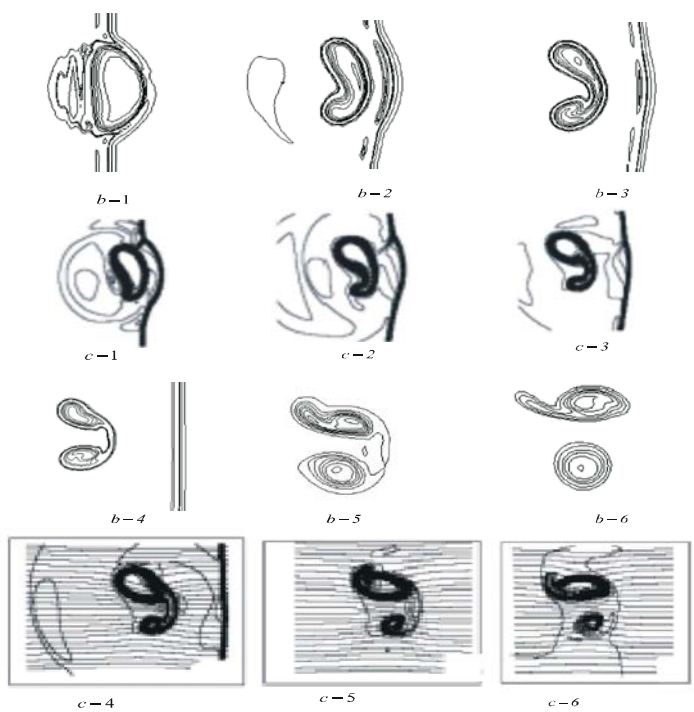

Fig. 6: Comparison the density contours related to grid 1000×200: (b-1, b-6) with adaptive unstructured algorithm by Povitsky *et al*. [10]: (c-1, c-6)

Table 2: The pressure comparison $(\frac{P}{P_{amb}})$ for different vorticity confinement methods and the grid 1000×200

Method	W/O CVC	Adaptive method	W/ CVC (=0.02)	W/ CVC (=0.2)	Povitsky Solution
Grid	1000-200	1000-200	1000-200	1000-200	Unstructured grid
MV	0.935	0.929	0.913	0.893	0.884
SV1	0.895	0.894	0.892	0.889	0.89
SV2	1.007	1.006	1.003	1.001	0.993

Table 3 The pressure comparison $(\frac{P}{P_{amb}})$ for different vorticity confinement methods and the grid 300×100

Method	W/O CVC	Adaptive method	W/ CVC (=0.02)	W/ CVC (=0.2)	Povitsky Solution
Grid	300×100	300×100	300×100	300×100	Unstructured grid
MV	0.969	0.958	0.946	0.936	0.884
SV1	0.954	0.931	0.902	0.895	0.89
SV2	1.008	1.007	1.005	1.003	0.993

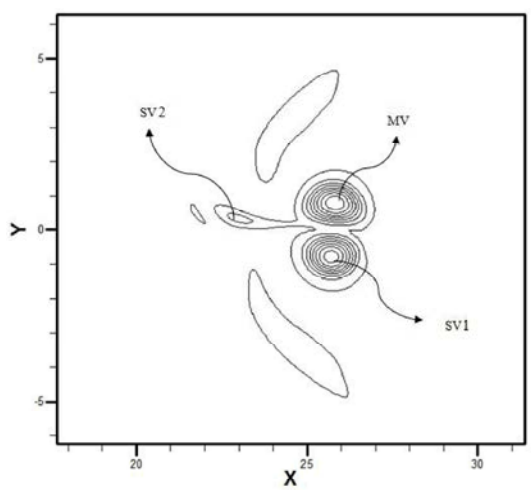

Fig. 7: Vorticity contours related to shock-vortex interaction at time t=421 μs.

are similar and they get closer to the adaptive unstructured algorithm results as the grid becomes finer (1000 × 200).

If we simulate the shock-vortex interaction using different vorticity confinement methods, the overall shapes for various steps of interaction are almost similar but they haven't shown here to prevent repetition. To compare the different vorticity confinement methods, it would be better to find the values of a variable (such as pressure) at the core of main and secondary vortices. Figure 7 shows the vorticity contours at time t=421 μs. Three different regions are specified in this figure: MV (Main vortex), SV1 (Secondary vortex 1) and SV2 (Secondary vortex 2).

Table 2 compares the pressure in the center of three vortices of the present study with different vorticity confinement methods and a fine grid *1000×200*. The results related to the coarser grid (300 ×100) are shown in Table 3. Also P_{amb} denotes the ambient pressure in these Tables.

The scalar vorticity confinement method with confinement parameter E_c, the new lambda confinement method with confinement parameter E_{cL} and the new adaptive method are compared in these tables. As we know, whatever the vorticity in the center of the vortex becomes larger, the pressure will be smaller, if the ambient conditions are fixed. Thus Tables 2, 3 show that the pressure in the center of three regions reduces with vorticity confinement (W/ CVC), relative to the without vorticity confinement (W/O CVC) state.

The most accurate method among three different vorticity confinement methods studied here, is the lambda method with E_{cL}=0.2 in tables 2, 3. Because the results of lambda method are closer to the results of Povitsky *et al*. [10] in contrast to the other vorticity confinement methods. As pointed earlier, the local conditions of flow field exist in the lambda formulation (according to Eq. (11)), thus this method has the best answer in contrast to the others. The results get better as the grid becomes finer *(1000×200)*.

Also the adaptive method is less accurate in contrast to the other vorticity confinement methods, but as pointed earlier its real worth is eliminating empirical constant coefficient (such as E_c, E_{cc}, E_{cL}). Thus this method is faster in contrast to the other vorticity confinement methods.

CONCLUSIONS

In this paper four different vorticity confinement methods (scalar, matrix, lambda and adaptive methods)

were used to solve three different vortex dominant problems. The most important conclusions of this paper are as the following:

- Different vorticity confinement parameters becomes more accurate when we consider the flow and computational parameters for them.
- The lambda vorticity confinement method has almost accuracy of the matrix method and its computational time is almost half of the matrix method. So this new method (lambda method) is useful enough.
- The adaptive vorticity confinement (with adaptive parameter) has less accuracy in contrast to the other vorticity confinement methods, but its real worth is eliminating empirical constant coefficients. So this method is faster than the other methods.

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