Middle-East Journal of Scientific Research 11 (11): 1546-1549, 2012
ISSN 1990-9233
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DOI: 10.5829/idosi.mejsr.2012.11.11.1633

Effect of Geometric Factor and Loading on Strength of Rectangular Plate Under Bending

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Abstract: In this research, effects of loadings and geometric factors such as thickness and length of rectangular plate on strength of plate under tensile load are investigated analytically. In this work, governing nonlinear differential equation will be employed for analyzing rectangular plate under direct applied loadings. Then, the mentioned equation would be analyzed by analytical method. Therefore, the maximum bending stresses (bending strength of plate) are obtained theoretically. Interestingly, suitable and logical results are found for the effects of geometric parameters and loadings on critical strength in rectangular plates.

Key words: Rectangular plates · Geometric factors · Strength of plate · Governing equation · Critical stresses

INTRODUCTION

In continuum mechanics, plates and shells theories are mathematical descriptions of the mechanics of flat plates which draws on the theory of beams. Plates are defined as plane structural elements with a small thickness compared to the planar dimensions. The typical thickness to width ratio of a plate structure is less than 0.1. A plate theory takes advantage of this disparity in length scale to reduce the full 3D solid mechanics problem to a 2D problem. Therefore, plates are initially flat structural elements, having thickness much smaller than the other dimensions. Included among the more familiar examples of plates are table tops, street manhole covers, side panels and roofs of buildings, turbine disks, bulkheads and tank bottoms. According to the criterion often applied to define a thin plate (for purpose of technical calculations) the ratio of the thickness to the smaller span length for rectangular plate should be less than 0.05. We assume that plate and shell materials are homogeneous and isotropic. Kirchhoff hypotheses are also considered in this research.

In recent years, theoretical analyses and various approaches have been analytically presented with purpose of obtaining suitable solutions and algorithms for analyzing nonlinear differential and ordinary equations [1-5]. For example, in reference [1], exp-function approach is employed to construct generalized solitonary solutions of the generalized regularized long-wave equation. In the mentioned reference, it has been shown that the exp-function method, with the help of symbolic computation, provides a simple and strong mathematical tool to solve nonlinear evolution equations with forcing term in mathematical physics.

Recently, traveling wave solutions, integral and iteration methods and such methods have been proposed for solving the nonlinear equations using various investigators [6-11]. For instance, it has been investigated the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity [9], in which, all explicit expressions of the bounded traveling wave solutions for the equation are obtained using bifurcation approach and qualitative theory of dynamical systems.

In addition, various investigations have been performed based on analytical and numerical methods for analyzing bending plates [12-18]. In this way, it has been described the summary of notch mechanics based on the linear elasticity [12], in which, the V-shaped notch with sharp or round corner and symmetric to the bisector is the object of the mentioned research.

It should be mentioned that, elastic stability [19, 20] and numerical simulation of composite plates [21] have been studied.

Also, other kinds of finite element and analytical methods have been investigated in some references [22-24]. It is noteworthy, comprehensive and perfect formulations and plate and shell's theories have been presented in references [25, 26].

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In this work, strength of rectangular plate is studied with simply supported edge boundary condition under applied uniform load. In which, effects of geometric factors such as thickness and plate dimensions and loadings on strength of rectangular plate will be analyzed theoretically. The theoretical results are interesting and logical.

In this investigation, governing nonlinear differential equations are employed for calculating maximum bending stresses in thin plate under applied load. By solving the governing nonlinear equation, we can obtain the strength of plate by direct method. The aim of this research is calculation of strength of rectangular plate with changes of geometric factors and applied uniform loadings.

MATERIAL AND METHODS

A rectangular plate subjected to uniform applied loading *p* is depicted in Figure 1 generally. A strip of unit width removed from a plate on this type will be in the same condition as a laterally and axially loaded beam or so-called tie-rod depicted in Figure 1. The value of the axial tensile forces N_x=N is such that horizontal movement of the edges is prevented.

In this article, it is assumed that a long rectangular plate with thickness *t* is bent into a cylindrical surface with its generating line parallel to the *y* axis. For this case $w = w(x)$ and the governing equation for deflection is given by,

$$w_{xxx} + 2w_{xxyy} + w_{yyyy} = D^{-1} \left(p + N_x w_{xx} + N_y w_{yy} + 2N_{xy} \right) \tag{1}$$

In which, *D* is flexural rigidity of the plate and defined as following,

$$D = \frac{Et^3}{12\left(1-\vartheta^2\right)} \tag{2}$$

Also N_x,N_y are loads in x, y directions respectively and N_{xy} is shear loading in x-y plane. In addition *p* is direct load. Equation (1) reduces to below form,

$$Dw_{xxx} = N_x w_{xx} + p \tag{3}$$

The bending moment at any section x of the strip is explained by

$$M\left(x\right) = \frac{1}{2} p_o x\left(l-x\right) - Nw \tag{4}$$

Equation (3) in term of this moment is given as below,

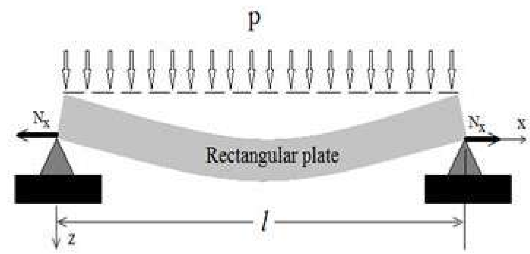

Fig. 1: Schematic of plate under load p.

$$Dw_{xx} = -M \tag{5}$$

Solution of above equation (5) with assuming proper boundary conditions $\left(w\,|_{x=0,l} = 0\right)$ is led to,

$$w = \frac{p_o D}{N^2} \left[\frac{\exp\left(0.5l\left(1-2xl^{-1}\right)\sqrt{ND^{-1}}\right) + \exp\left(-0.5l\left(1-2xl^{-1}\right)\sqrt{ND^{-1}}\right)}{\exp\left(0.5l\sqrt{ND^{-1}}\right) + \exp\left(-0.5l\sqrt{ND^{-1}}\right)} \right] \tag{6}$$

The maximum deflection and moment $\left(\frac{dw(x)}{dx} = \frac{dM\left(x\right)}{dx} = 0\right)$ happens at $\left(x = \frac{1}{2}\right)$, that is,

$$w_{\max} = \frac{p_o D}{N^2} \left[\frac{2}{\exp\left(0.5l\sqrt{ND^{-1}}\right) + \exp\left(-0.5l\sqrt{ND^{-1}}\right)} + \frac{N^2 l^4}{32D^2} - 1 \right] \tag{7}$$

$$M_{\max} = \frac{poDl^2}{Nl^2} \left[1 - \frac{2}{\exp\left(0.5l\sqrt{ND^{-1}}\right) + \exp\left(0.5l\sqrt{ND^{-1}}\right)} \right] \tag{8}$$

It should be noted that if there were no tensile reactions at the ends of the strip, the maximum deflection and moment would be,

$$M_{max} = 0.125 p_o l^2 \tag{9}$$

In then follows that the maximum bending moment is given by equation (8) and maximum bending stress (bending strength) is given by,

$$\sigma_{bending(x)} = \frac{6M_{\max}}{t^2} = \frac{0.75 p_o l^2}{t^2} \quad (10)$$

RESULTS AND DISCUSSIONS

To examine the validity of the present analytical method, the steel is chosen as a test case. At last, acceptable results are determined analytically for obtaining critical stresses in order to present geometric effects and loadings on strength of plate. As mentioned before, in this paper, material is assumed steel for verifying the results of analytical method. Governing differential equation is solved by direct method generally. In the next sections, the critical stresses will be calculated in rectangular plates subjected to uniform load with assumed boundary conditions (Simply supported edges) by direct method.

Observe that the calculation of the plate displacement simplifies to the solution of equation (3), which is of the same from as the differential equation for deflection of beams under the action of lateral and axial forces. If plate edges are not free to move horizontally, a tension in the plate is produced depending upon the magnitude of lateral deflection w. The problem then becomes intricate. The tensile forces in the plate carry part of the lateral loading through membrane action.

Figures 2, 3 show effects of geometric factors and related loadings on strength of plate (maximum bending stresses) subjected to bending moments and lateral loads. As expected, gradients of bending stress curves increase with increasing the geometric factor $\frac{l}{t}$ and load p. Therefore, bending strength of plate intensively depends on geometric factors in rectangular plates generally.

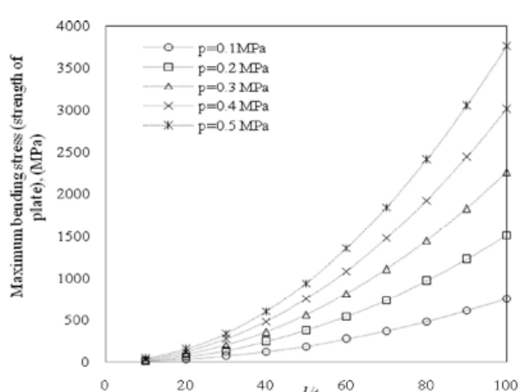

Fig. 2: Effect of geometric factors and loading on maximum bending stress (strength of plate).

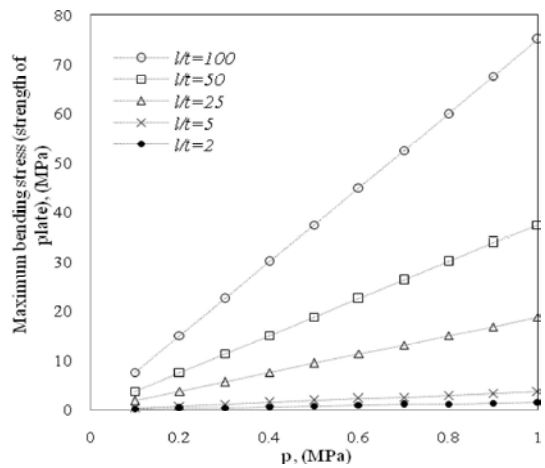

Fig. 3: Effect of geometric factors and loading on maximum bending stress (strength of plate).

CONCULSIONS

The governing differential equation was solved with assumed deflections by analytical direct method that boundary conditions were satisfied by the mentioned deflections. Sectional geometry of the rectangular plate was closed generally. Also the boundary conditions were simply supported edges. With obtaining maximum moment in plates, bending strength and maximum stress in rectangular plates will be analytically determined simply. Accordingly, maximum bending stress and strength of plate depends on geometric factors and related loads in rectangular plates logically. Furthermore, gradients of bending stress curves decrease with decreasing the geometric factor $\frac{l}{t}$ and load p.

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