Analysis of the Effects of Mixing Height and Other Associated Factors on the Effective Dispersion of Plume

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(Received: March 19, 2010; Accepted: February 25, 2011)

Abstract: The overall focus of the research work was to study the various factors that affect plume dilution and dispersion. Some of the factors that were studied include; the effects of mixing height, the effects of plume rise and the effects of terrain in addition to momentum and buoyancy on the overall dispersion of plume released from a stack of known effective height. Data on temperature versus altitude was collected using an infrared thermometer at different height of a telecommunication mast under construction. The highest temperature for the month was noted and the validity of the recorded data was done using correlation analysis. Mathematical analysis was then employed to determine the mixing depth which represents the effective height of any stack that must be placed in such location in order to allow for complete dispersion/dilution of any form of pollutant released from any source. Result obtained shows that the effective height of stack that can be erected in such location that will allow for effective dispersion of any pollutants was shown to be 1700m. Any stack below this height will lead to ground level pollution. Also discussed in this research paper is the application of Gaussian Plume model in the evaluation/analysis of the horizontal dispersion of pollutants released from a height (h).

Key words: Plume Dispersion, Mixing Height, Momentum/Buoyancy, Gaussian Plume equation, Air quality

INTRODUCTION

Emissions of pollutants into the atmosphere can take place in different forms; For example, wind blows dust into the air. When plant material decays, methane is released. Automobiles, trucks and buses emit pollutants from engine exhausts and during refueling. Electric power plants along with home furnaces give off pollutants as they try to satisfy mankind's need for energy. In all of these, the release of pollutants from stack has received more attention from all others. Stacks come in all sizes from a small vent on a building's roof to a tall stack. Their function is to release pollutants high enough above the earth's surface so that emitted pollutants can sufficiently disperse in the atmosphere before reaching ground level. Taller stacks disperse pollutants better than shorter stacks because the plume has to travel through a greater depth of the atmosphere before it reaches ground level. As the plume travels it spreads and disperses effectively if not under the influence of aerodynamic and other factors. The behaviour of a plume emitted from an elevated source such as tall stack depends on the degree of instability of the atmosphere and the prevailing wind turbulence. Plume rise analysis and modeling has found real time applications in stack design and location that will promote effective dispersion of pollutants and eliminate ground level air pollution concentration. In air pollution studies, ground level concentration of pollutants is of paramount importance, hence the need for serious evaluation of factors and parameter that tends to enhance the effective dispersion/dilution of any form of pollutant released from any source. This research work is geared towards evaluating such parameters that will allow for effective dispersion/dilution of pollutants thus preventing...
ground level concentration of the pollutants and hence protecting the environment from irregular ground level pollutant concentration.

**Analysis of the Effects of Mixing Height on Plume Dispersion**

**Research Methodology:** The methodology of research involves collection of temperature versus altitude data in a vertically upward manner. An infra red thermometer was used to record temperature across a mast of vertical height 45 meters, at 5 meters interval in other to understand the variability of temperature with height. Three different temperature data were collected for each height and the mean temperature value was taken as the actual temperature at that point. Data collection was done in the month of March, 2009. The highest temperature value during the period under study was also recorded.

Validation of the data collected was done using statistical analysis by computing the coefficient of correlation at 99% confidence interval and (n - 1) degree of freedom. The need for validation was to determine the degree to which one can depend on the data collected.

To determine the nature of the atmosphere around the study area, two temperature data and the corresponding altitude value was used to compute the ambient lapse rate (ALR) which is normally used to classify the different types of atmosphere viz; stable, unstable and neutral atmosphere. The whole idea is to understand the behaviour of the plume once it is released from the stack and also the stability of the atmosphere, since atmospheric stability has a direct influence on plume dilution and dispersion.

The mixing height which reflect the effective height that will allow for total plume dispersion thus preventing ground level pollutant concentration was determine by plotting a graph of altitude versus the corresponding temperature taken into consideration the highest temperature of the day.

**RESULT AND DISCUSSION**

Pollutants once released into the atmosphere, are largely affected by the degree of instability of the atmosphere. Atmospheric stability on the other hand, is normally affected by the variation of temperature with altitude. This variation affects the vertical mixing / dispersion of the pollutants. Mathematically, this variation is define by the linear relationship between the Dry Adiabatic Lapse Rate [DALR] and the Ambient Lapse Rate [ALR]. Whereas, dry adiabatic lapse rate (DALR) is the reference indicator based on the assumption that temperature decreases with altitude, having an actual variation of 1\(^{\circ}\) reduction in temperature for every 100m, the ambient lapse rate (ALR) is obtained practically, by measuring the vertical temperature with altitude using selected measuring instruments. Mathematically, (ALR) is given as \(\frac{Z_2 - Z_1}{T_2 - T_1}\) (2). Where \(T\); is the measured temperature and \(Z\) is the corresponding altitude in meters.

The table below shows the result of temperature versus altitude values.

The highest temperature for the month was gotten to be 34\(^{\circ}\)C. Statistical hypothesis involving test of coefficient of correlation was employed to ascertain the validity of the data collected and the table below shows the result of the analysis.

The coefficient of correlation \((r)\) was then calculated to be [0.14]. At 99% confidence interval, with a degree of freedom equals \((n - 1)\), the critical value was found to be [0.92]. Since the value of \((r)\) was less than the critical value, we conclude that their is a slight linear variation between temperature and altitude. This conclusion is correct since the effects of wind speed and wind velocity profile cannot be neglected. Hence data are certified correct and can be used for modelling. The model is aimed at understanding the nature of atmosphere around the vicinity of the mast and to predict the behaviour of any pollutant that will be emitted around such location.

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<th>20</th>
<th>30.0</th>
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<th>Altitude (m)</th>
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<td>10</td>
<td>29.5</td>
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<td>30</td>
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Table 1: Temperature Vs Altitude

Table 2: Coefficient of Correlation (r)

Table 3: Atmospheric Stability
For this model, two temperature readings and the corresponding altitude were used to compute the value of the Ambient Lapse Rate [ALR], the resultant value was then multiplied by 100 and plotted on a linear graph to ascertain the nature of the atmosphere and the corresponding stability at that location, result is shown below;

\[
ALR = \frac{dT}{dz} = \frac{T_2 - T_1}{Z_2 - Z_1} = \frac{31.5 - 29.5}{30 - 10} = 0.1
\]

\[
ALR = 100 \times 0.1 = 10
\]

This value was then represented on a linear graph as shown below:

**Unstable Atmosphere Stable Atmosphere:** The model reveals that the location from where the data was collected is a stable atmosphere (Fig. 1). The implication of this, is that any pollutant release in that location will be suppressed and cannot be dispersed effectively thus causing serious pollution problem around such vicinity. The only solution to this problem is to determine the mixing depth, that is, the effective height of any stack that must be placed in such location that will allow for the effective dispersion and dilution of the pollutants. The result of (Fig. 10) shows that the effective height of stack that can be erected in such location that will allow for effective dispersion of any pollutants is shown to be 1700m. Any stack below this height will lead to ground level pollution. Model of this kind will enhance the effective mixing and dilution of pollutants and should be strictly adhered to during the design and construction of plume stack by petrochemical industries.

**Evaluation of Associated Factors Affecting Plume Dispersion:** Some of the associated factors affecting plume dispersion/dilution are discussed below;

**Plume Rise and its Effects on Plume Dispersion:** Gases that are emitted from stacks are often pushed out by fans. As the turbulent exhaust gases exit the stack they mix with ambient air. This mixing of ambient air into the plume is called entrainment. As the plume entrains air into it, the plume diameter grows as it travels downwind. These gases have momentum as they enter the atmosphere. Often these gases are heated and are warmer than the outdoor air. In these cases the emitted gases are less

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**Fig. 1: Conning Plume**

**Fig. 2: Looping Plume**

**Fig. 3: Fanning Plume**

**Fig. 4: Fumigating Plume**

**Fig. 5: Lofting Plume**
Fig. 6: Wind Speed Affects Pollutants Dispersion

Fig. 7: Stable Atmosphere (ALR < DALR)

Fig. 8: Unstable Atmosphere (ALR > DALR)

Fig. 9: Neutral Atmosphere (ALR = DALR)

dense than the outside air and are therefore buoyant. A combination of the gases’ momentum and buoyancy causes the gases to rise. This is referred to as plume rise and allows air pollutants emitted in this gas stream to be lofted higher in the atmosphere. Since the plume is higher in the atmosphere and at a further distance from the ground, the plume will disperse more before it reaches ground level. Some other behaviour observed when plume is emitted from stack are discussed below:

**Conning Behaviour:** A conning plume can occur under cloudy sky during day and night when the lapse rate is essentially Neutral (Fig. 6). This condition of the atmosphere will neither suppress nor enhance dispersion of the plume. The plume shape is vertically symmetrical about the plume center line. A major part of the pollutant is carried downward fairly far before reaching the ground level.

**Looping Behaviour:** This kind of plume behaviour occurs under super adiabatic condition i.e. unstable condition where large scale thermal eddies is present which support turbulence and dispersion. This eddy carry portion of the plume to the ground level for short time periods causing momentary high surface concentration of pollutant near the stack. The unstable nature will cause a kind of turbulences thus making the plume to move in a zig – zag direction (Fig. 7).

**Fanning Behaviour:** This kind of plume behaviour occurs under condition of stable atmosphere. For a high stack, fanning is considered a favourable meteorological condition because it does not contribute to ground level pollution (Fig. 8).

**Fumigating Behaviour:** Conning, looping and fanning occur under condition of uniform lapse rate. But when the lapse rate changes from stable to unstable as in the case when an inversion condition is broken in the early hours the morning. In this case, a stable layer of air lies a short distance above the point of plume release and an unstable layer lies below. This unstable layer of air causes the plume to mix downwards towards the ground (Fig. 9).

**Lofting Behaviour:** It is the inverse of those of fumigation. Here an unstable layer of air lies above the point of source of the plume while a stable layer of air lies below the plume. Lofting is the most favourable plume type as far as ground level concentration is concerned (Fig. 10).

**Effects of Terrain on Plume Dispersion:** The terrain of the environment can affect the effective dispersion of the pollutants. Cases like heat island, land and sea breeze interface, valleys and hilltops have been discussed.
Heat Islands: In urban areas where there exist so much building, heat is absorbed and re-radiated at high rate which imposes vertical convection currents on the already prevailing meteorological condition making the atmosphere much more unstable. Thus, plume is likely to reach the ground more rapidly due to turbulence or rapid mixing.

Land and Sea Breezes: In the land area closer to the sea, currents are set up as a result of alteration of day and night. During the day, the land gets heated up faster than the sea but at night the land also cool faster than the sea. Thus, at night, the cool air moves from land to sea which due to buoyancy rises and set up a circulation. As such, at night, the interface between land and sea creates an inversion making plume release to be of fanning nature.

Mountains and Hill: The presence of mountain will disturb the flow of air in the horizontal directions and thus the dispersion of pollutants can be affected by the presence of mountains. Usually, high level ground level concentration of pollutants will be found close to stacks located in mountain regions as a result of aerodynamic down wash.

Building Wake: Sometimes, plume release from stack can get trapped behind a building preventing effective dispersion leading to high level pollutants concentration. Building wake is caused by wind flowing over building that causes low pressure wake behind the building. The wake-effects evaluation procedures may be applied by the user to any stack on or adjacent to a building. For regulatory application, a building is considered sufficiently close to a stack to cause wake effects when the distance between the stack and the nearest part of the building is less than or equal to five times the lesser of the height or the projected width of the building. For downwash analyses with direction-specific building dimensions, wake effects are assumed to occur if the stack is within a rectangle composed of two lines perpendicular to the wind direction, one at 5Lb downwind of the building and the other at 2Lb upwind of the building and by two lines parallel to the wind direction, each at 0.5Lb away from each side of the building. A simple way to avoid building wake is to build stack at a level 2.5 times the actual height of the tallest building around the stack location.

Effects of Momentum and Buoyancy on Plume Dispersion: The condition of the atmosphere, including the winds and temperature profile along the path of the plume, will largely determine the plume's rise. The plume characteristics influence plume rise. The exit velocity of the exhaust gases leaving the stack contributes to the rise of the plume in the atmosphere. This momentum carries the effluent out of the stack to a point where atmospheric conditions begin to affect the plume. Once emitted, the initial velocity of the plume is quickly reduced by entrainment as the plume acquires horizontal momentum from the wind. This causes the plume to bend over. The greater the wind speed is the more horizontal momentum the plume acquires. Wind speed usually increases with distance above the earth's surface. As the plume continues upward the stronger winds tilt the plume even further. This process continues until the plume looks level may be a considerable distance downwind from the stack. Wind speed is important in blowing the plume over. The stronger the wind, the faster the plume will tilt over. Plume rise due to its buoyancy is a function of the temperature difference between the plume and the surrounding atmosphere. In an atmosphere that is unstable, the buoyancy of the plume increases as it rises, increasing the ultimate plume height. In an atmosphere that is stable, the buoyancy of the plume
decreases as it rises. Finally, in a neutral atmosphere, the buoyancy of the plume remains constant. Buoyancy is taken out of the plume by the same mechanism that tilts the plume over the wind. As shown below, mixing within the plume pulls atmospheric air into the plume interior. The faster the wind speed is, the faster this mixing with outside air takes place. Entrainment of ambient air into the plume by the wind "robs" the plume of its buoyancy very quickly so that on windy days the plume does not climb very high above the stack (Fig. 11).

**Estimation of the Horizontal Dispersion of Plume:**

Plume rise analysis and modeling provide no information about how pollutant concentrations vary from the centerline. Dispersion estimates must be made in order to determine pollutant concentrations at a point of interest. Dispersion estimates are determined by using distribution equations and/or air quality models. These dispersion estimates are typically valid for the layer of the atmosphere closest to the ground where frequent changes occur in the temperature and distribution of the winds. These two variables have an enormous effect on how plumes are dispersed. When a plume leaves a stack, it must possess sufficient momentum and buoyancy so as to continue to rise from the stack exit. When there is no wind, low density plume tend to reach high elevation and ground concentration are usually low. It is observed that when a plume originates from a height \( h \), it rises additional height \( \Delta h \) rising to the buoyancy of the hot gases with a vertical velocity \( V_s \). Thus for practical purposes, the plume appears as if it originates from a point source at an equivalent stack height \( H = h + \Delta h \) where \( H \) is the effective stack height which is normally employed for modeling (Fig. 11).

For a model describing the whole process above, it is assumed that a plume travelling horizontally in the \( x \) direction at a mean speed \( u \) disperses horizontally \( y \) and vertically \( z \) so that the concentration of the pollutant at any cross section of the plume following the normal Gaussian probability distribution, thus for any point \( x, y, z \) in the plume, the concentration \( C \) of the pollutant is given by

\[
C_{x,y,z} = \frac{Q}{2\pi\rho\delta y\delta z} \exp\left(-\frac{1}{2}\frac{y^2}{\delta y}\right) \left[\exp\left(-\frac{1}{2}\frac{z-H}{\delta z}\right)^2 + \exp\left(-\frac{1}{2}\frac{z+H}{\delta z}\right)^2\right]
\]

Where

- \( C_{x,y,z} \) : Concentration of pollutants at point \( x, y, z \) (\( \text{UG/m}^3 \))
- \( x \) : Distance directly downwind (m)
- \( y \) : Horizontal distance from plume center line (m)
- \( z \) : Vertical distance (m)
- \( Q \) : Emission rate of pollutants (Kg/s)
- \( H \) : Effective stack height (m)
- \( U \) : Average wind speed at effective stack height (m/s)
- \( \delta y \) : Horizontal dispersion coefficient (m)
- \( \delta z \) : Vertical dispersion coefficient (m)

In air pollution studies, we are normally interested in what happen at the ground level; hence to use this model to study ground level pollution, we may need to impose \( z = 0 \) that is we neglect the vertical distance effects. The plume size is dependent on the stability of the atmosphere and the dispersion of the plume in the horizontal and vertical directions. These horizontal and vertical dispersion coefficients (\( \sigma_y \) and \( \sigma_z \) respectively) are merely the standard deviation from normal on the Gaussian distribution curve in the \( y \) and \( z \) directions. These dispersion coefficients, \( \sigma_y \) and \( \sigma_z \), are functions of wind speed, cloud cover and surface heating by the sun. The Gaussian distribution requires that the material in the plume be maintained. In other words, the plume edge must be allowed to reflect from the ground without losing any pollution. In addition, the Gaussian distribution and plume rise depend on the ground being relatively flat along the path of the plume. In order for a plume to be modeled using the Gaussian distribution the following assumption must be made:

- The plume spread has a normal distribution
- The emission rate \( Q \) is constant and continuous
- Wind speed and direction is uniform
- Total reflection of the plume takes place at the surface

Due to the configuration of the stack or adjacent buildings, the plume may not rise freely into the atmosphere. Some aerodynamic effects due to the way the wind moves around adjacent buildings and the stack can force the plume toward the ground instead of allowing it to rise in the atmosphere. For point source pollution at ground level, the basic assumptions are as follows: No cross wind effects \( (y = 0) \), pollutant source at ground level \( (H = 0) \) and pollutant has reached ground level in which case \( z = 0 \). Imposing these assumptions on the generalized Gaussian plume equation, we have:
For point source pollution at an elevation (H) above the ground, the basic assumptions are as follows: No cross wind effects (y = 0), pollutant has reached ground level (z = 0) and pollutant release is at a height H above the ground (H ≠ 0). Imposing these assumptions on the generalized Gaussian plume equation, we have:

\[ C_{x,0,0} = \left[ \frac{Q}{21\mu \delta y \delta z} \exp\left\{ \frac{1}{2} \left( \frac{0 - H}{\delta z} \right)^2 \right\} \right] \exp\left\{ \frac{1}{2} \left( \frac{0 - (H - y)}{\delta z} \right)^2 \right\} \]

For maximum ground level concentration from elevated source with cross wind effect, the basic assumptions are as follows: Presence of cross wind effects (y ≠ 0), pollutant has reached ground level (z = 0) and pollutant release is at a height H above the ground (H ≠ 0). Imposing these assumptions on the generalized Gaussian plume equation, we have:

\[ C_{x,y,0} = \left[ \frac{Q}{21\mu \delta y \delta z} \exp\left\{ \frac{1}{2} \left( \frac{y - y^*}{\delta y} \right)^2 \right\} \right] \exp\left\{ \frac{1}{2} \left( \frac{0 - (H - y)}{\delta z} \right)^2 \right\} \]

\[ \Delta h = \frac{V_d}{u} \left[ 1 + \frac{2.68 \times 10^{-2} (P)}{T_s - T_a} d \right] \]

Where

\[ V_d = \text{Stack velocity, velocity at which the plume leaves the top of the stack (m/s)} \]
\[ d = \text{Stack diameter (m)} \]
\[ u = \text{Wind speed (m/s)} \]
\[ P = \text{Atmospheric pressure (Kpa)} \]
\[ T^\circ C = \text{Stack temperature (K)} \]
\[ T_a = \text{Ambient temperature (K)} \]
\[ \Delta h = \text{Plume rise height (m)} \]
\[ H = h + \Delta h \]

The Holland equation is functional at all stability; Neutral, Stable and Unstable. One major requirement is that the pressure must be known. If the pressure (P) is not known, then the plume rise height \( \Delta h \) can be calculated using the bricks equation

\[ \Delta h = 2.6 \left( \frac{F}{U_a S} \right)^{1/3} \]

Where

\[ F = \text{The buoyancy flux parameter (m}^4\text{s}^{-3}) \]
\[ S = \text{Stability parameter (S}^2\text{)} \]
\[ U_a = \text{Wind speed at stack height (m/s)} \]

The buoyancy flux parameter \( F \) can be determined from

\[ F = gr^2 V_s \left[ 1 - T_s/T_a \right] \]

Where

\[ g = \text{Acceleration due to gravity (m/s}^2\text{)} \]
\[ r = \text{Radius of the stack (m)} \]
\[ V_s = \text{Stack gas exit velocity (m/s)} \]
\[ T_s = \text{Stack gas temperature} \]
\[ T_a = \text{Ambient temperature (K)} \]

The stability parameter \[ S \] is calculated as follows:

\[ S = \frac{g \Delta T_u}{T_a \Delta Z} + 0.010C/m \]

Where

\[ \Delta T_u/\Delta Z = \text{Actual Ambient lapse rate (}^\circ\text{C/m)} \]
\[ g = \text{Acceleration due to gravity (m/s}^2\text{)} \]
\[ T_a = \text{Ambient temperature (K)} \]
The bricks equation is functional for stable condition [Stability class E – F].

If the pressure is not known and the stability condition is Neutral or Unstable i.e. stability class A – D, then the real equation applies:

$$\Delta h = \frac{1.6 \left( \frac{F^3}{x_f} \right)^{2/3}}{U_n}$$

Where

$$F = gr^2V_s [1 - T /T_s]$$

Note:

If $F \geq 55 m^3/s$ then, $x_f = 120 F^{0.4}$

If $F < 55 m^3/s$ then, $x_f = 50 F^{0.8}$

**Air Quality Modelling:** The dispersion models require the input of data which includes:

- Meteorological conditions such as wind speed and direction, the amount of atmospheric turbulence (as characterized by what is called the "stability class"), the ambient air temperature and the height to the bottom of any inversion aloft that may be present.
- Emissions parameters such as source location and height, source vent stack diameter and exit velocity, exit temperature and mass flow rate.
- Terrain elevations at the source location and at the receptor location.
- The location, height and width of any obstructions (such as buildings or other structures) in the path of the emitted gaseous plume.

**REFERENCES**


