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Kamal Al-Din Al-Farisi's Explanation of the Rainbow

Hüseyin Gazi Topdemir

Department of History of Science, Faculty of Letters, Ankara University, Ankara, Turkey

Abstract: What is dealt with in this article is a critical presentation of the arguments put forward by Kamal al-Din al-Farisi about formation of the rainbow. This optical phenomenon was explained by two scientists, an Eastern one, Kamal al-Din al-Farisi and a Western one, Theodoric of Freiberg, simultaneously but independent from each other. And what is more interesting is that their explanations of rainbow were nearly correct in some respects and somewhat similar to our present understanding. This study, especially, reveals that Kamal al-Din al-Farisi is well ahead of his time in his assumptions related to most of the above mentioned topic.

Key words: Rainbow . Aristotle . Kamal al-Din al-Farisi . Theodoric of Freiberg . Halo . Kitab al-Manazir . Ibn Sina . Tanqih al-Manazir

INTRODUCTION

The explanation of the formation of the rainbow was one of the foremost problems in the optical science during the middle ages, both in the East and the West. This optical phenomenon was explained by two scientists, an Eastern one, Kamal al-Din al-Farisi (d. 1320) and a Western one, Theodoric of Freiberg (1250-1311), simultaneously but independent from each other. What is more interesting is that their explanations of rainbow were nearly correct in some respects and somewhat similar to our present understanding.

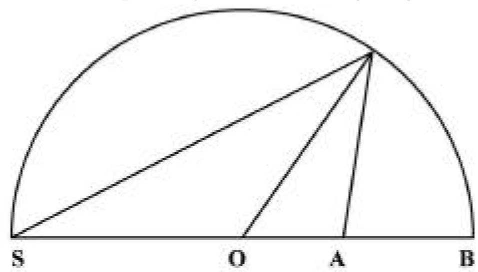
As it is reported in the literature **[1]** various explanations of rainbow go back to very old times, although some of them may be considered rather speculative or mythological. For instance, the Ancient Germans thought that the rainbow was a bridge for gods to take a trip around the world and also Ancient Japans Shinto priest had same ideas about it. For Babylonian, rainbow is the necklace of love goddess Ishtar. Similarly, in the Ancient Chinese literature, it was found that there were various classifications of the rainbow, used to predict future. According to Ancient Chinese people, rainbow was a synthesis of these principles: Yang, the masculine principle, Yin the feminine principle. Speculations about the nature of the rainbow were lacking among the ancient Greeks. In the famous epic of Homer, in Iliad the goddess Iris takes to Aphrodite from the battle area to Olympus by following the rainbow.**[2]**

Rainbow as a scientific problem appears to have been treated first by Aristotle in the history of optics. Although the explanations of Aristotle can to no longer be considered to be tenable, yet they are very important in view of the great influence which they exercised upon the later development of the subject. Thus, the study of rainbow has become a subject of discussion frequently, both in the Christian and in the Islamic World.**[3]**

Aristotle knew both the causal relation between the existence of particles and the formation of rainbow and the geometrical relation among the relative positions of the sun, the observer and the arc. These are the

Corresponding Author: Dr. Hüseyin Gazi Topdemir, Department of History of Science, Faculty of Letters, Ankara University, Ankara, Turkey

two important steps leading to a complete explanation of the rainbow.**[4]** In the opinion of Aristotle the rainbow comes into existence in a hemisphere, the centre of which is the observer's eye and the base of which is the horizontal line, Fig. 1. He believed in what he called "meteorological sphere" with dense clouds inside. All his explanations were based on this belief. For him, there are three main elements necessary for the formation of rainbow, namely, the source of light, the observer and the dense clouds. The formation depends on the different positions of these three elements. That is, the rainbow occurs after reflection from the dense clouds in the "meteorological sphere", if the rays of sunlight reach the observer's eye. This is the primary rainbow. Sometimes two independent rainbows may occur simultaneously in the sky. In this case, the farther one is the secondary rainbow and the colours are much pale than those of the primary one, since its distance is longer; the colour of the primary rainbow being brighter because of its proximity.



S = Sun, O = Observer, C = Cloud

Fig. 1: Aristotelian explanation of the rainbow

So far, I have tried to sum up Aristotle's understanding of rainbow. And his ideas on this subject cannot be supported today. Indeed, as we know, rainbow occurs when the rays of sunlight are refracted twice in a raindrop and are reflected once from it, Fig. 2 and the secondary rainbow is formed after another reflection following the first one. This reflection causes the colour to be pale and to be inverted, Fig. 3. Aristotle, however, speaks of a meteorological sphere and tells us that the rainbow is formed by reflection from the cloud in the sphere at a limited distance from the observer. Here he doesn't mention refraction. He was also mistaken, when he says: The secondary rainbow is pale because it is farther off than the primary.**[5]** It is also a contradiction, because he already assumed that the sphere and the cloud were at a limited distance from the observer. Now, the observer is at the centre of the sphere, irrespective of the positions of the sun and the clouds they should be at equidistant from the observer and therefore the change in the distance would not happen.

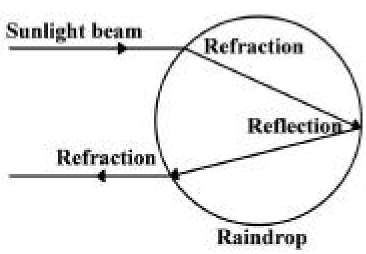


Fig. 2: The formation of the primary rainbow

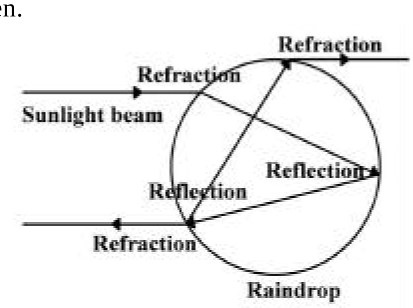


Fig. 3: The formation of the secondary rainbow

Despite Aristotle's incorrect understanding, his explanations of rainbow were effective for centuries and most popular in the Islamic World. For instance, Ibn Sina's study of rainbow is not much different from Aristotle's. To Ibn Sina, rainbow is formed as a result of the reflection of light from the small transparent dewdrop particles dispersed in wet air rather than in the cloud.**[6]** We can say that Ibn Sina's only success was that he gave relatively less importance to the role of the cloud, which was very important in Aristotle's account of the rainbow. And the idea of using the dew instead of the cloud provided him with the possibility to examine the phenomenon geometrically. Unfortunately, Ibn Sina did not succeed either.**[7]** His explanations of the secondary rainbow are not coherent. For him, the light at higher levels, being much closer to the sun, is reflected more strongly, so the red colour is formed. Accordingly, the outermost arc of the secondary rainbow must be red. However, it is violet. This indicates that Ibn Sina's explanation on the formation of the secondary rainbow is wrong. But his general observations**[8]** on the problem are significant with respect to the fact that they provide more knowledge about the topic.

Certainly, Ibn Sina is not the only one who studied this subject in the Islamic World. One of his contemporaries, Ibn al-Haytham (965-1039), who has been accepted as the greatest scholar of optics of all times and was also called as the second Ptolemy **[9]** carried out successfully refraction experiments and extensive studies on the subject.**[10]**

Ibn al-Haytham treated the formation of rainbow in an article called *Maqala fi al-Hala wa qaws quzah*.**[11]** In this article he explained the formation of rainbow as an image, which forms at a concave mirror. If the rays of light coming from a farther light source reflect to any point on axis of the concave mirror, they form concentric circles in that point.**[12]** When it is supposed that the sun as a farther light source, the eye of viewer as a point on the axis of mirror and a cloud as a reflecting surface, then it can be observed the concentric circles are forming on the axis.

In this treatise Ibn al-Haytham's explanation of the rainbow fails, being conceived of solely in terms of reflection from a concave surface formed by cloud. He, therefore, concluded that the rainbow is formed as a result of the reflection from the cloud. Although it is a different approach, it does not contribute much to the problem. Whether the cloud is plain or concave, it is not significant for the correct understanding, since the approach is merely based on reflection.

As it can easily be seen, Ibn al-Haytham made no significant contribution to the problem of the formation of the rainbow. However, his optical studies in general and particularly his success in geometrical optics had a great influence on his successors. In fact, Ibn Rushd (1126-1198) is the one who was clearly influenced by his studies. In his explanation of the formation of rainbow, he roughly repeated Ibn al-Haytham's concept of concave surface.**[13]** But Ibn al-Haytham's treatise becomes one of the starting points of Kamal al-Din al-Farisi's more successful researches.

Another scientist who is worth mentioning in the process of a right explication of the rainbow and who seemed to have been influenced by the studies of Ibn Sina and Aristotle is al-Qarafi (d. 1283). He studied the conditions required for the formation of the rainbow and established the relative positions of the sun, the observer and the arc. According to his view, the rainbow is formed as a result of reflection of the sunlight from the water vapour in the air.**[14]** Although he did not mention the clouds, his explanations about the formation of the rainbow are based on reflection. Therefore, his studies are important only in the sense that they provide a link with other studies, which in turn led to the correct explanation of the rainbow.

Other scholars who studied this problem in the Islamic World are Nasr al-Din al-Tusi (d. 1275), Qutb al-Din al-Shirazi (1236-1311) and Kamal-Din al-Farisi. Al-Shirazi almost correctly explained the formation of the rainbow, though how he did it is not known clearly. However, it is possible to get some clues in this connection from his book on astronomy, called *Nihayat al-Idrak*.**[15]** Another source is the book by his pupil Kamal al-Din al-Farisi, with the title *Tanqih al-Manazir*. In this book he sometimes uses the phrase

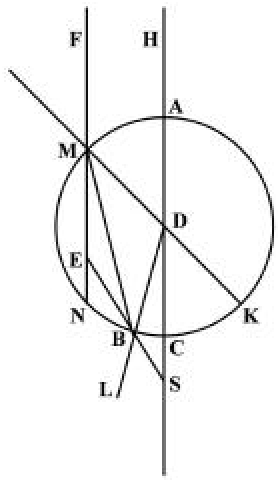


Fig. 4: Refraction of the ray from a transparent sphere

"we say" and sometimes "I say". It is not unreasonable to identify "we" by himself and his teacher al-Shirazi and "I" may be taken for himself alone. Kamal al-Din al-Farisi is in fact the one scholar who made significant contributions to the problem of the formation of rainbow.

Kamal al-din al-farisi's explanation of the rainbow: In fact, Kamal al-Din al-Farisi did not write a separate book on the formation of rainbow. But we can have information about his studies from his *Tanqih al-Manazir* which is a commentary on Ibn al-Haytham's *Kitab al-Manazir*. In this commentary book Kamal al-Din al-Farisi dealt with Ibn al-Haytham's work on Burning Spheres.**[16]** There, Ibn al-Haytham had postulated some principles for Burning Spheres Kamal al-Din al-Farisi try to interpret them.

1. A ray coming to a sphere, as parallel to its axis, but outside of it (The point S on the Fig. 4).**[17]**
2. The angel formed by the ray deviated from the sphere to the point outside the sphere (to point S) and the axis (HDS) is as twice as the deviation angle (BMN).**[18]**
3. Rays coming to the surface of the sphere which are getting deviated from the axis reach beyond the first point (point S).**[19]**
4. Only one ray reaches the point S.**[20]** Rays coming to the sphere as parallel to its axis, from an inverted cone. The top of this cone is the burning centre of the sphere of which distance to the sphere is a quarter of the diameter of sphere.

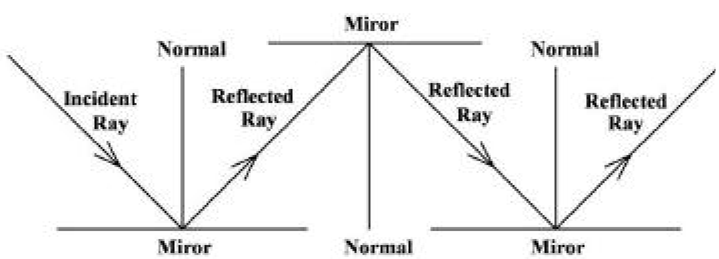


Fig. 5: Multi–reflection of a ray

He adds nothing to the first three principles but interprets them. He just tries to make them clearer so that they can be understood. If we examine the text, it will be seen that he says nothing as a contribution to Ibn al-Haytham's first three principles. The contribution that Kamal al-Din al-Farisi made to the fourth principle of Ibn al-Haytham's is that he uses a term "ending point" for the points where rays disappear. But he makes his major contribution to the fifth principle. There he makes the problem clearer by determining it and taking out the useless repetition and making correction on the wrong values. And he also determines the place where rays converged.

In addition, he successfully deals with the changes occurring when rays pass through a low dense medium from a denser medium. And he gives us a table of the refraction.**[21]**

After determining the changes taking place when light passes through the burning spheres and the points where burning occurs, he starts his study of rainbow, using the results he has obtained. In the beginning of his study, he states that there are four ways of obtaining the images by means of a bright and transparent sphere.**[22]** Here the sphere he mentions could be a glassy sphere but filled with water or completely natural raindrop or a dew particle as well.

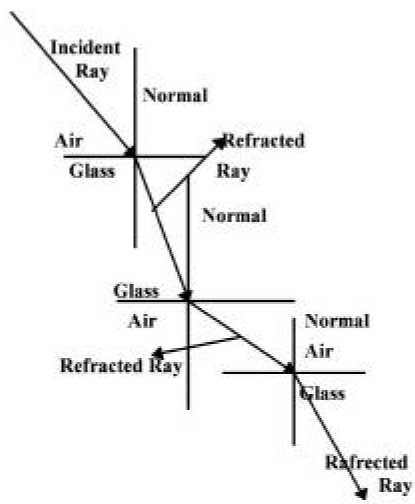


Fig. 6: Multi – refraction of a ray

According to Kamal al-Din al-Farisi, when the sunrays fall on a reflective or refractive surface, they reflect from or refract to another point. If there is another reflective (Fig. 5) or refractive (Fig. 6) surface, they will continue reflection or refraction. This may happen several times. But through these processes the structure of the ray never changes but remains the same.**[23]**

When a transparent sphere is placed in front of an eye, a cone occurs with the axis of a straight line between eye and the surface in front of it. Rays coming from the axis pass through the sphere without changing the direction, that is, they do not deviate, but the others deviate because of density of the sphere. The angle of incidence becomes wider correlating to the length if the distance from the top of the sphere. Finally, it becomes 90 Fig. 7.**[24]**

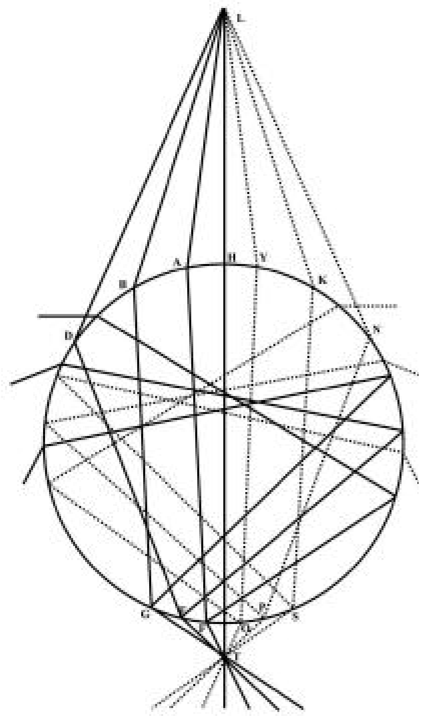


Fig. 7: The paths of rays in a transparent sphere

He successfully determined the paths, which the rays, emerging from a source, follow through. So rays come to the sphere with certain angles. Among these rays, those that are closer to the axis pass through it in a nearer point. This intersection occurs outside the sphere completely. The rays in the right side of the sphere deviate to the left and vice versa. Namely LA, LB, LC and LD in Fig. 7 represent rays coming from the right and LY, LK, LM and LN from the left. Thus the ray LA on the right deflects in the sphere to AI, LB to BG, LC to CF and LD to DE. Correspondingly the ray LY on the left deflects to YO in the sphere, LK to KS, LM to MR and LN to NP.**[25]**

Kamal al-Din al-Farisi successfully showed the change occurred when the sunlight enters the sphere and found how many times each ray reflects and refracts, in the light of the information he had obtained before. Thus he shows that rays undergo 1) only two refractions, 2) two refractions and one reflection, 3) two reflections and two refractions. This is a totally correct account. Accordingly, it can be possible to see this in Fig. 7 given by him through various simplifications. For example in Fig. 8 the ray moving from L, after falls the surface of the sphere, will penetrate into the sphere due to transparency of the sphere and at the same time it will undergo a refraction due to differences in the density of medium and it will come to E, following the DE way. And again the same ray will leave the sphere T as result of its transparency and it will refract again.

But the ray coming to E does not refract completely. In the interior part of the sphere, i.e. in the raindrop, functioning as a concave mirror, a little part of the ray reflect, E in Fig. 8. In other word, the formerly refracted ray coming to E will reflect at this point in addition the former one. And it will arrive at K, following the EK way. So the ray will undergo two refractions and one reflection Fig. 9.

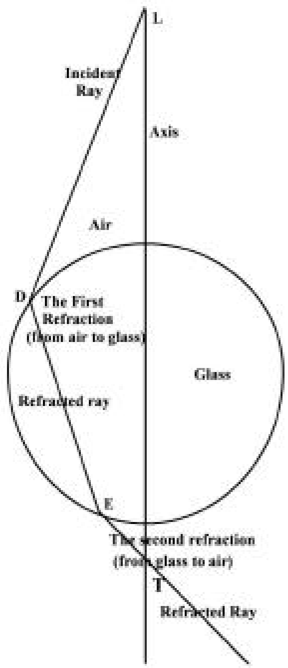


Fig. 8: Double refraction of a ray in a transparent sphere

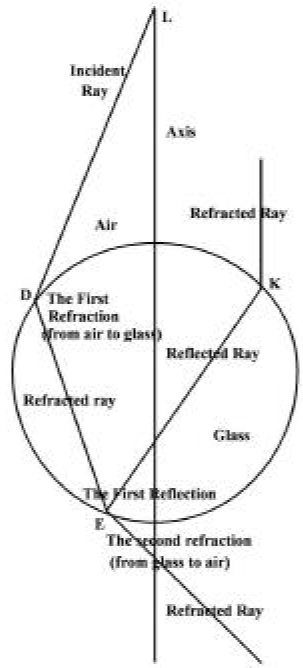


Fig. 9: Double refraction and one reflection of a ray

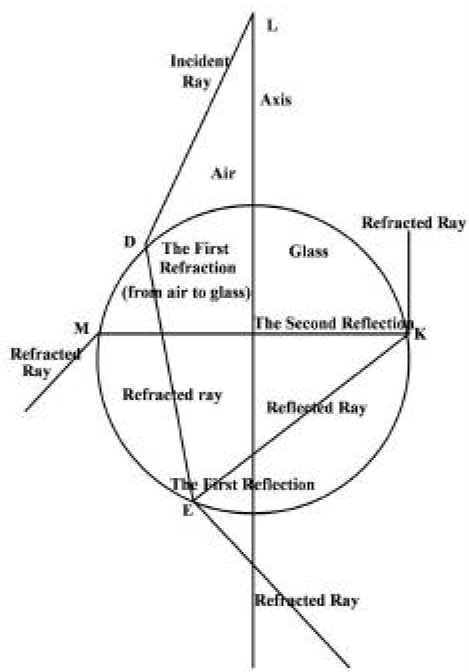


Fig. 10: Double refraction and double reflection of a ray

As the density of the sphere remains the same i.e. it is a homogenous body, the ray arriving at K will be subjected to two changes. In other words, because of the transparency of the sphere it will refract and, because of the brightness of sphere it will reflect. Rays reflecting from K will arrive at M, but this time following the KM path inside of sphere. Here, two refractions and reflections occur Fig. 10.

Indeed, rays at M point are subjected to third process of refraction and reflection because of their nature. But it is not easy to determine this third refraction and reflection because it is getting weak and weak after so many processes of refraction and reflection. Here the first thing to be pointed out is that Kamal al-Din al-Farisi gives us a complete explanation of the primary rainbow, in his investigation about light alterations, because, as it is know, the primary rainbow takes form as a result of two refractions and one reflection of the sunlight in the water droplets. So, I think we can say that Kamal al-Din al-Farisi was able to explain the formation of primary rainbow as similar to today's understanding. And, inevitably his third determination will explain the secondary rainbow because the secondary rainbow appears as a result of a process consisting of two refractions and reflections. That is to add the second reflection to the first one. As a result, the order of the colour of the secondary rainbow will be the opposite the primary rainbow.

Kamal al-Din al-Farisi's third success is related to the third reflection, as it has been mentioned above, since this determination will answer whether a third rainbow occurs. As it is known Aristotle explained the paleness of the secondary rainbow basing on a totally false theory. For Aristotle, as the secondary rainbow appears farther than the primary one, its colour seems to be more pale because of the longer distance. However, Kamal al-Din al-Farisi based his theory on the number of refractions and reflections. On the other hand, Ibn Sina accepted the possibility of a third one, but he could not explain how the brightness of colour get weaker. Whereas, Kamal al-Din al-Farisi could explain why the third one does not occur and why the colour get weaker. Thus, with Kamal al-Din al-Farisi, the Islamic World gained a great success in optics.

Later, Kamal al-Din al-Farisi experimentally investigated the image created by refractions and reflections say, rainbow. He placed a transparent glass sphere in a dark in such a way, that the sun light beams enter only through a hole on the sphere and he investigated the structure of it. In this experiment he saw that the image got bigger as the sphere was brought nearer to the light source but in reverse, smaller and finally disappeared.**[26]**

CONCLUSIONS

Kamal al-Din al-Farisi's experiments were repeated by Descartes (1596-1650) when he got interested in rainbow.**[27]** Accordingly, when he studied the problem, he observed that rainbow occurred as a result of spreading of water from a sprinkler and thus he concluded that water spread consisted of small water droplets. So he made some experiments in the light of the sun, with glassy sphere full of water. Standing on foot and directing his back to the sun, he watched through a hole in the glassy sphere, shaking it upward and downward, he finally discovered brightness at the bottom of the sphere. Kamal al-Din al-Farisi made similar experiments and obtained the same results as Descartes, many years before him. On the other hand, it is know that many years later, Newton (1642-1727) utilized prisms to obtain colour**[28]** and he investigated how colour appear and disappear depending on the prism's distance to light source. It is interesting that as he was carrying out his experiment he used a dark room lighted through only a small hole, just as Kamal al-Din al-Farisi did.

Meanwhile, Kamal al-Din al-Farisi's success was also achieved by Theodoric, who was his contemporary in the West. In the second part of his treatise he wrote on this subject. Theodoric says, "When the light falls on a single raindrop, it refracts twice and reflects once before it reaches the observer's eye.**[29]** Following this, remark, when he made relations between the rainbow and the raindrop, he states that "the sunlight falls on the top of water droplet, it refracts and enters the sphere and then it falls on the interior concave surface and reflects back. Finally, it refracts and reaches our eyes".**[30]** No doubt, these are correct determinations about the problem, the formation of the rainbow. And Theodoric also explained the formation of the secondary rainbow in a fully correct way, by considering that it is the result of two refractions and reflections.**[31]**
Now, there are two questions to be replied.

1. Did the two influences each other?
2. Who first reached the correct explanation Kamal al-Din al-Farisi or Theodoric?

Referring to the first question, it cannot be surely said that there was a direct influence of one on the other. Because Theodoric's death date and Kamal al-Din al-Farisi's teacher Qutb al-Din al-Shirazi's death date are the same. And Theodoric wrote his book between 1304 and 1310. On the other hand, Farisi tells us that when he wrote his book *Tanqih al-Manazir*, he had great help from his teacher. So his ideas must have developed in same period as Theodoric's. For these reasons we should consider that they developed their theories in the same period but independent from each other.

What is then the reason for this simultaneous success? As it is known, the detailed explanation of the rainbow was not made until the end of the 13^{th} century.

Here we should speak of one of the most striking aspects of science that is the fact that scientific knowledge is cumulative. Naturally by then, there had been a great amount of accumulation of knowledge in terms of quantity and quality relating to the problem. Particularly the greatest success in the geometrical optics was achieved by Ibn al-Haytham. He made it possible to use the optics in geometrical studies and therefore, more sophisticated explanations were also made possible. Now, the secret of their success relies on Ibn al-Haytham's book, *Kitab al-Manazir*. As a matter of fact, while Kamal al-Din al-Farisi's (d. 1320) book is commentary on Ibn al-Haytham's (965-1039) book, Theodoric, (1250-1311) too, emphasizes in his own book *De Iride*, that he had obtained the information he needed (especially about refraction) from *Kitab al-Manazir*.**[32]** So, we can say that the secret of the simultaneous of their success lies on using the same sources but independent from each other. But they went further and made great contributions to optics and gave us a correct explanation of the rainbow, same as understand.

Finally, it would be said that Kamal al-Din al-Farisi gained a great success as his explanation of the rainbow is the same as today's understanding, because, he was able to explain the formation of rainbow basing on a raindrop. This led him to achieve a great success, which many others could not. As we know, the interior parts of a raindrop functions as a medium. Thus, when the sunlight falls on a raindrop, they both undergo refraction and reflections and as a result, the rainbow occurs. In addition, Kamal al-Din al-Farisi did not investigate the rainbow only in the sky, but he also carried it to the laboratory to study it in detail.

Footnotes

1. Carl B. Boyer, *The Rainbow, from myth to mathematics*, (New Jersey 1987), pp. 17-32.
2. Boyer, p. 18; see also, Aydin Sayili, "The Aristotelian Explanation of the Rainbow", *Isis*, 1940, 30:65-83, on p. 65.
3. Sayili, "The Aristotelian", p. 65.
4. Sayili, "The Aristotelian", p. 65.
5. Sayili, "The Aristotelian", p. 70; Aydin Sayili, "Light, Vision and Rainbow in Ibn Sina", *Ibn Sina* (in Turkish), (Ankara 1984):203-241, pp. 236-237.
6. Boyer, *The Rainbow*, p. 78.
7. For instance, he could have deduced from his observations that the rainbow is a complete circle, if observed at higher elevations. See Sayili, "Light", p. 240., also Boyer, *The Rainbow*, p. 78.
8. Boyer, The Rainbow, p. 80.
9. George Sarton, *Introduction to the History of Science*, Vol. I, (Baltimore 1927), p. 721.
10. This text was completed in A.D. 1028 (A.H. 419) and critically edited by Kamal al-Din al-Farisi in *Tanqih al-Manazir*, II, pp. 258-279. A shortened German translation of this edition is; Eilhard Wiedemann, "Theorie des Regenbogens von Ibn al-Haiṯam", in *Sitzungsberichte der Physikalisch-Medizinischen Sozietät in Erlangen*, 1914, 46:39-56. The original Arabic text is in Süleymaniye Library, Atif Efendi Collection number 1714, (12 folios) in Istanbul.
11. Wiedemann, "Theorie", pp. 50-51; Joseph Würschmidt, "Die Theorie des Regenbogens und des Halo bei Ibn al-Haiṯam und bei Dietrich von Freiberg", *Meteorologische Zeitschrift*, 1914, 31:484–487, p. 485.
12. Boyer, *The Rainbow*, p. 83.
13. Aydin Sayili, "Al-Qarafi and his Explanation of the Rainbow", *Isis*, 1940, 32:14-26, p. 16; See also Boyer, *The Rainbow*, p. 126.
14. George sarton, *Introduction to the History of Science*, Vol. II, (Baltimore 1929), p. 1012.
15. Boyer, *The Rainbow*, pp. 127-128.
16. *Maqala fi al-Kura el-Muḫriqa* (On the Burning Sphere) written after *Kitab el-Manazir*. The text critically edited by Kamal al-Din al-Farisi in *Tanqih al-Manazir*, II, 285-302. A German translation of this text is Eilhard Wiedemann, "Brechung des Lichtes in Kugeln nach Ibn al Haiṯam und Kamal al Din al Farisi", in *Sitzungsberichte der Physikalisch – Medizinischen Sozietät in Erlangen*, 1910, 42:15-58. A Turkish translation and commentary of same text is Hüseyin Gazi Topdemir; *Kamal al-Din al-Farisi's work on the Burning Sphere*, unpublished Master's Thesis, Faculty of Letters, Ankara University, (1987).
17. Al-Farisi, *Tanqih*, pp. 285-286.
18. Al-Farisi, *Tanqih*, pp. 287-288.
19. Al-Farisi, *Tanqih*, pp. 288-290.
20. Al-Farisi, *Tanqih*, pp. 291-292.
21. Al-Farisi, *Tanqih*, p. 302.
22. Al-Farisi, *Tanqih*, p. 299.
23. Al-Farisi, *Tanqih*, p. 303.
24. Al-Farisi, *Tanqih*, p. 304.
25. Al-Farisi, *Tanqih*, pp. 316-317.
26. Al-Farisi, *Tanqih*, pp. 317-319.

27. A.I. Sabra, *Theories of Light, from Descartes to Newton*, (London 1967), pp. 61-62.
28. Isaac Newton, *Opticks*, Book I, Part I, Proposition II, Theorem II, (London 1931), p. 26.
29. Boyer, *The Rainbow*, p. 114; see also Joseph Würschmidt, "Die Theorie des Regenbogens und des Halo bei Ibn al-Haiṯam und bei Dietrich von Freiberg", *Meteorologische Zeitschrift*, 1914, 31:484 – 487.
30. Boyer, *The Rainbow*, p. 115.
31. Boyer, *The Rainbow*, p. 117.
32. Boyer, *The Rainbow*, p. 125.

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