Number of Physical Degrees of Freedom in Constrained Hamiltonian Systems

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Abstract: We give a simple deduction of the Díaz-Higuita-Montesinos formula for the number of physical degrees of freedom in terms of Lagrangian parameters.

Key words: Hamiltonian systems · Singular Lagrangians · First class constraints · Gauge identities

INTRODUCTION

Díaz-Higuita-Montesinos [1, 2] deduced the following expression to obtain the number of physical degrees of freedom (NPDF) in systems with singular Lagrangians:

\[ \text{NPDF} = N - \frac{1}{2} (l + g + e), \] (1)

in terms of Lagrangian parameters, in fact, \( N, e, l \) and \( g \) are the total number of generalized coordinates \( q(t) \), effective gauge parameters \([1, 3, 4]\), genuine constraints and gauge identities \([5-10]\), respectively.

This same calculation can be realized via the Hamiltonian formula \([3]\):

\[ \text{NPDF} = N - N_1 - \frac{1}{2} N_2, \] (2)

employing only concepts from the Rosenfeld-Dirac-Bergmann approach \([11-19]\) (also see Bronstein \([20]\) and Haag \([21]\)), where \( N_1 \) and \( N_2 \) are the total number of first-and second-class constraints \([3, 5, 15, 22-24]\), respectively; let’s remember that \( N_2 \) is an even number \([3, 15, 25]\) and that the number of degrees of freedom is the same for Hamiltonian and Lagrangian formalisms \([26]\).

In \([26, 27]\) was established the following relation:

\[ l = N_1 + N_2 - N_1^{(p)}, \] (3)

being \( N_1^{(p)} \) the total number of first-class primary constraints; besides \([3]\):

\[ g = N_1^{(p)} = M - \text{rank}(\{\varphi_j, \varphi_m\}). \] (4)

where \( M \) is the amount of independent primary constraints.

If we accept that the Dirac’s conjecture \([3, 5, 16, 18, 26, 28-30]\) is valid, then the \( N_1 \) first-class constraints (primary and secondary) generate gauge symmetries into the Hamiltonian formalism in accordance with the number of effective gauge parameters in the Lagrangian process \([3]\):

\[ e = N_i, \] (5)

thus, from (3), (4) and (5) we have that \( N_1 = e, N_2 = l + g - e \) & \( N_1^{(p)} = g \), therefore (2) implies the formula (1) obtained by Díaz-Higuita-Montesinos \([1]\).

In \([4]\) was showed how to apply (1) to several Lagrangians studied in \([5, 31-33]\).

REFERENCES


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