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Optimization of Reservoir Volume by Yield Model and Simulation of it by Dynamic Programming and Markov Chain Method

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Abstract: Because of lack or shortage of hydrometric data, these data must be produced by a suitable method. This method should memorize characteristics of stochastic parameters. The Markov chain model makes used of correlation between data and can memorize characteristics of stochastic parameters. The arrival flow the reservoir of dam has stochastic characteristics. Discharges of arrival flows were produced by Markov chain method in this research. Optimum volume of reservoir of dam was calculated by yield model. Yield model made used of produced discharges by Markov chain method and required water for downstream of dam. Yield model considers critical year for optimization of volume of reservoir of dam. Discharge of inflow to reservoir is the least value in critical year. Results of yield model are confident for different conditions. The number of deficits (the number of months that released flow from reservoir is less than required water for downstream of dam.) was calculated by a simulation program. Simulation program made used of dynamic programming in this research. Several scenarios were considered for yield model and simulation program. In these scenarios, the necessary drinkable water was prepared perfectly while required water for farms was prepared from 60% to 70%. For evaluation of this research, the Amirkabir dam on the Karaj River in Iran was considered.

Key words: Markov chain method · Yield model · Dynamic programming · Drinkable water demand · Irrigation water demand · The Amirkabir dam

INTRODUCTION

The volume of water is a constant value in the world. But the population of the world and their required water increases rapidly. Preparation of required water will become a very important problem in near future. Therefore water resource management is an important knowledge for creation an advanced society. A scientific subject in water resource management is optimization of the volume of reservoir dams. By optimization of the volume of reservoir dams, the required water of the downstream of dams will be prepared economically and the volume of reservoir dams will become minimize. Dams must reserve water in wet periods in order to they can provide required water of their downstream in drought periods. The long term variation of discharge of inflows to reservoir and required water of the downstream of dam must be considered for optimization of volume of reservoir dam. Water demands are several types as drinkable water, required water for irrigation of farms, required water for hydropower plants and required water for environmental necessaries. Discharge of inflows to reservoir has stochastic characteristics. The continental variation, climatic variation and human activity are important factors that can vary discharge of inflows to reservoir very much. By attention to occurrence of extreme drought at recently years in Iran, new time series of discharge of inflows to reservoir must be produced by the Markov chain method. The Markov chain method is a stochastic method that produces new time series of data abased on available time series of data. This method can memorize characteristics of stochastic parameters. After production of time series of data, the time series that show drought must be selected for optimization of volume of reservoir dam. A number of researchers produced time series of different data by the Markov chain method as:

Scientists applied the Markov chain method for prediction of drought periods in the Goksu River in Turkey [1]. Researchers utilized the Markov chain Monte Carlo method for updating of parameters of a sediment

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entrainment model [2]. Also a number of researchers compared the Markov chain Monte Carlo method and the Halphen distribution. They observed the Markov chain Monte Carlo method can produce hydrologic data better than the Halphen distribution because variance and skew ness coefficient of produced data by the Markov chain Monte Carlo method is similar to actual values of these parameters [3]. Several engineers applied the Markov chain method for simulation and management wastewater system in San Diego city. They predicted transition probabilities over the approaches developed in the past, including the nonlinear optimization-based approach, in terms of versatility in the implementation, precision of the estimated data and appropriateness of the assumptions in the model [4]. Scientist made used of the hybrid Markov chain method for generation of daily discharges of the Tisza River and its tributaries. The Tisza River pours to the Danube River in Hungary [5].

A number of researchers applied dynamic programming for simulation and optimization of volume of reservoir dams as:

Researchers applied linear programming (LP), nonlinear programming (NLP) and dynamic programming (DP) for solution of problems in water resource management [6, 7]. Two researchers made used of combination of (LP) and (DP) for optimization of volume of parallel multi objective reservoirs. Their case study was Shasta and Folsom dams in California Valley project [8]. Scientists developed a model for operation a multi objective reservoir. Their case study was Shasta dam in California [9]. Also other researchers combined (LP) and (DP) methods for solving a system with 22 decision variables. They utilized small time steps (hourly and daily) in central valley project [10].These researchers applied this method in different projects [11, 12]. Two engineers applied (DP) method for determination of the value of required water in future [13]. Recently researchers developed folded dynamic programming (FDP) method. This method is applied for optimization of multi reservoirs systems. This method does not need to primary path for finding of global optimum. Therefore this method does not converge to local optimums. Also the number of iteration of this method is less than the number of iteration of dynamic programming for reaching to global optimum [14]. Researchers extracted monthly optimization scenarios for Hoover reservoir in central Ohayo. They made used of dynamic- regression programming and linear programming with chance restriction. The results of operation scenarios of two methods were compared by simulation methods. Mean yearly damage of operation scenarios of linear programming with chance restriction was less than mean yearly damage of operation scenarios of dynamic-regression programming [15]. Also researchers made used of (DP) for optimization of multi objective reservoirs [16].

These researchers paid attention to generation of synthetic data or optimization and simulation of volume of reservoir dams. But the most of researchers did not consider combination of two subjects. In this research, two subjects were combined by researchers. For evaluation of this study, the Amirkabir dam on the Karaj River was selected.

The Amirkabir Dam: The Amirkabir dam was constructed on the Karaj River in 1961. The area of its watershed is 764 Km^2. The average of yearly discharge of inflow to its reservoir is 472 MCM. This dam locates is 63 Km west north of Tehran and 23 Km north of Karaj city. This dam supplies a part of drinkable water of Tehran and water for irrigation demand of 50000 hectares of farms near to Karaj city. Also its hydropower plant can produce 90 MW electrical energies. This dam is a two arches concrete dam. The height of dam from bottom, the length of crest of dam and the width of crest of dam are 160m, 390m and 8m respectively. The total volume of reservoir dam is 202 MCM. The bottom elevation of reservoir and normal water surface elevation of reservoir are 1545m and 1610 m respectively. The volume of useful storage of reservoir dam is equal to 191.6 MCM. The position of the Amirkabir dam in Iran and its watershed are shown in Figure 1.

The Research Methodology: This research was accomplished in three steps:

Step1: Generation of synthetic data by the Markov chain method:

Model that generates synthetic data must reserve characteristics of main data. Main data have serial correlation. Serial correlation is shown by r(k) (k is time step). Data of each month have serial correlation with data of last month and data of next month. The equation between two dependent data is:

$$Y=a+bX \quad (1)$$

If $r^2=1$, the value of observed data (Y_0) will become equal to calculated value of data by equation1 (Y).

r^2: Correlation coefficient between Y_0 and X

Because of $r^2<1$, Y can not become equal to Y_0. Difference between Y and Y_0 is shown by e.

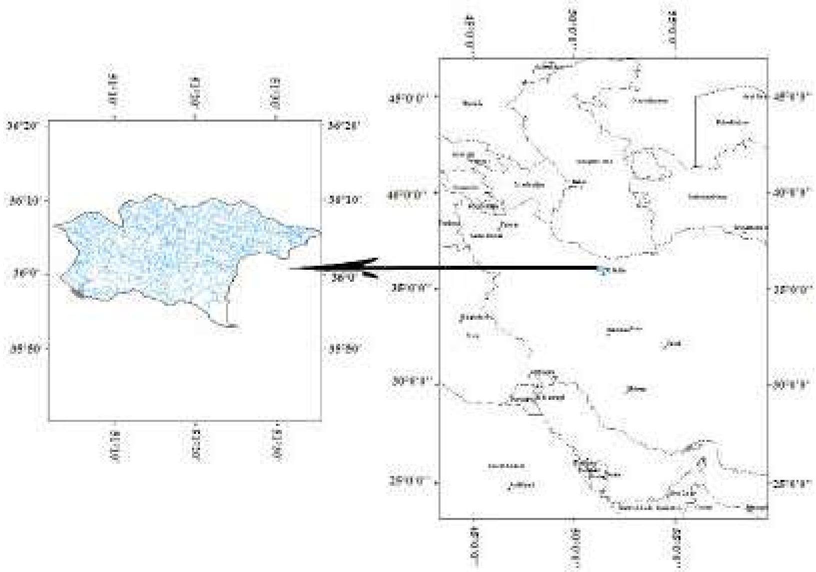

Fig. 1: The position of the Amirkabir dam in Iran and its watershed
Fig. 1: The local of the Amirkabir dam in Iran

$$Y_0=Y+e=a+bX+e \tag{2}$$

The mean of e is equal to zero. The value of e is determined by Chow formula.

$$e=\bar{e}+K_T S_e \tag{3}$$

Where:
S_e: Standard deviation of e
Because of $\bar{e}=0$, $e=K_T S_e$.

By attention to $r^2<1$ the equation2 has two parts. These parts are deterministic part (a+bX) and stochastic part (e). By assumption $t=K_T$, equation2 converts to equation4.

$$Y_0=a+bX+tS_e \tag{4}$$

Because Y_o is summation of e and a constant value, governing stochastic distribution on it is similar governing stochastic distribution one. Also t depends on governing stochastic distribution on e. The values of (a) and S_e are determined by equations5 and 6.

$$a=\overline{Y_0}-b\overline{X} \tag{5}$$

$$S_e=S_{Y0}\left(1-r^2\right)^{1/2} \tag{6}$$

Where:
$\overline{Y_0}$: The mean values of Y_0
$\overline{X}$: The mean values of X
S_{Y0} : The standard deviation values of Y_0

By attention to equations (5) and (6), equation (2) converts to:

$$Y_0=\overline{Y}_0+b(X-\overline{X})+tS_{y0}\left(1-r^2\right)^{1/2} \tag{7}$$

The value of (b) is calculated by equation8:

$$b=r\frac{S_{y0}}{S_X} \tag{8}$$

In this research, it made used of a seasonal Markov chain method with time step one month. The equation of this method is:

$$X_{i,j+1}=\overline{X}_{j+1}+b_j(X_{i,j}-\overline{X}_j)+t_{i,j+1}S_{X_{j+1}}\left(1-r_j^2\right)^{1/2} \tag{9}$$

Where:
I: Index of year
j: Index of season or month or week or day
r_j: Correlation coefficient between data of month j and data of month j+1
b_j: Regression coefficient that is calculated by equation10.

$$b_j = r_j \frac{S_{j+1}}{S_j} \qquad j = 1,\ldots,\omega \tag{10}$$

ω: The number of months

If $j = \omega \rightarrow j+1 = 1$, $X_{i,j+1} = X_{i+1,1}$, $t_{i,j+1} = t_{i+1,1}$,

$$b_j = r_j \frac{S_{X1}}{S_{X\omega}} \quad and \quad b_\omega = r_\omega \frac{S_{X1}}{S_{X\omega}}.$$

The value of $t_{i,j+1}$ is calculated by governing stochastic distribution on data. If governing stochastic distribution is normal distribution or log normal distribution, the value of $t_{i,j+1}$ will become equal to the value of parameter z of normal distribution. But if governing stochastic distribution is Pearson III distribution or log Pearson III distribution, $t_{i,j+1}$ will be calculated by equations11 and 12.

$$Cs_{e_j} = \left(Cs_{Xj} - r_{j-1}^3 Cs_{Xj-1}\right) \Big/ \left(1 - r_j^2\right)^{1.5} \tag{11}$$

$$t_{i,j+1} = \frac{2}{C_{se_{j+1}}}\left[1 + \frac{C_{sej+1}k_{i,j+1}}{6} - \frac{C_{sej+1}}{36}\right]^3 - \frac{2}{C_{sej+1}} \tag{12}$$

Where:

C_{sej}: Skew ness coefficient of the values of e in month j

$K_{i,j+1}$ is equal to the value of parameter z of normal distribution.

Step2: Optimization of volume of reservoir of the Amirkabir dam by yield model:

Yield model considers critical year for optimization of volume of reservoir. The volume of inflow to reservoir in critical year is less than volume of inflow to reservoir in the other years. Therefore results of yield model are confident for design and planning. The determined volume of reservoir by yield model can often supply required water of downstream of dam. While determined volume of reservoir by other models may not supply required water of downstream of dam. In the other hand yield model considers only a year while a perfect model must consider total years of time series. This subject decreases the number of decision variables and restrictions considerably. Yield model assumes that summation of yearly volume of inflow to reservoir at each year is equal to summation of yearly volume of inflow to reservoir in critical year (Y_f). Also yield model assumes that monthly distribution of volume of inflow to reservoir in each year is similar to monthly distribution of volume of inflow to reservoir in critical year. Monthly distribution of volume of inflow to reservoir in critical year is shown by β_j.

$$\sum \beta_j = 1 \tag{13}$$

$$\sum \beta_j Y_f = Y_f \tag{14}$$

Continuous equation for critical year is shown by equation15.

$$S_j + \beta_j Y_f - Y_{fj} = S_{j+1} \qquad \forall_j \tag{15}$$

Where:

S_j: Storage of reservoir in the start of month j

S_{j+1}: Storage of reservoir in the end of month j

$\beta_j Y_f$: The volume of inflow to reservoir in month j

Y_{fj}: Water demand in month j

For determination of β_j, it makes used of mean monthly distribution of volume of inflow to reservoir in critical year and the fifth critical year.

$$\beta_j = \frac{q_{j_c}}{Q_c} \tag{16}$$

Where:

q_{jc}: The volume of inflow to reservoir in month j of critical year

Q_c: The volume of inflow to reservoir in critical year

Yearly water demand has several components as drinkable water, water demand for irrigation, water demand for hydropower plant and etc. Drinkable water should be prepared ceaselessly. This component is named primary demand. But continuous preparation of other components is not necessary. These components are named secondary demands. Secondary demands are supplied occasionally. The probability of preparation of secondary demands is calculated by equation17.

$$P = \frac{(n-f)}{n+1} = \frac{n_s}{n+1} \Rightarrow n_s = (n+1)P \tag{17}$$

Where:

f: The number of deficits (the number of months that released water from reservoir in them is less than water demand in the downstream of dam.)

n: The number of years

n_s: The number of successful years

P: The probability of preparation of secondary demands

For determination of n_s, the one-zero programming must be applied. The one-zero programming is an integer programming. The coefficient of the one-zero programming is α(α=1 for successful years α=0 for deficits). Relation between α and ns is shown by equation18.

$$\sum_{i=1}^{n} \alpha_i = n_s \tag{18}$$

Monthly distribution of primary demands and secondary demands are shown by equations19 to 22.

$$y_{firmt} = \wp_{ft} * y_{firm} \tag{19}$$

$$y_{pt} = \wp_{pt} * y_p \tag{20}$$

$$\sum_{t=1}^{T} \wp_{ft} = 1 \tag{21}$$

$$\sum_{t=1}^{T} \wp_{pt} = 1 \tag{22}$$

Where:

$\wp_{ft}$: Monthly distribution coefficient of primary demands

$\wp_{pt}$: Monthly distribution coefficient of secondary demands

Y_{firmt}: Monthly primary demands

Y_{pt}: Monthly secondary demands

Y_{firm}: Yearly primary demands

Y_p: Yearly secondary demands

Objective function in yield model= min Ka (23)

Restrictions of yield model divide to two parts:

Yearly restrictions:

ST:

$$S_{i+1} \le S_i + Q_i - Y_{firm} - E_i - \alpha_i y_P \quad \text{I=1 to n} \tag{24}$$

$$E_i = \left[a + b\left(S_i + \sum_{t=1}^{T} \frac{S_t + S_{t+1}}{2} * \wp_t \right) \right] * E_m \tag{25}$$

$$S_{n+1} = S_1 \tag{26}$$

$$S_i \le K_a^0 \quad \forall_i = 1,\dots,n \tag{27}$$

Where:

Ka: Total storage of reservoir

S_i: Storage of reservoir in the start of year i

S_{i+1}: Storage of reservoir in the end of year i

Q_i: The volume of inflow to reservoir in year i

E_i: The volume of evaporation from reservoir in year i

a, b: Constants that are calculated by volume- area curve of reservoir.

$\wp_t$: Monthly distribution coefficient of evaporation

S_t: Storage of reservoir in the start of month t

S_{t+1}: Storage of reservoir in the end of month t

E_m: The yearly height of evaporation from reservoir

K_a^0: Yearly storage of reservoir

Monthly restrictions:

ST:

$$S_{t+1} \le S_t + \beta_t \left[y_{firm} + y_P + E_i \right] - Y_{firmt} - e_t - y_{Pt} \tag{28}$$

$$e_t = \left[a + b\left(\frac{S_t + S_{t+1}}{2} \right) \right] e_{mt} \qquad \forall_t = 1 \dots T \tag{29}$$

$$S_{T+1} = S_1 \tag{30}$$

$$S_{\min} + K_a^0 + S_t \le K_a \qquad \forall_t = 1,\dots,T \tag{31}$$

Where:

e_t: The volume of evaporation from reservoir in month t

e_{mt}: The height of evaporation from reservoir in month

tS_{min}: The dead storage of reservoir

Step3: simulation of preparation of required water in the downstream of dam by dynamic programming:

Simulation controls released water from reservoir. If released water from reservoir is more than water demand, the addition volume of water should be reserved in reservoir for preparation of required water in next months. If released water from reservoir is less than water demand, the volume of released water from reservoir will increase in order to it become equal to water demand. The time step of simulation model was one month in this research. The storage of reservoir in the start of each time step was state variable of system and released water from reservoir in each time step was decision variable of system. The method of solution was backward dynamic programming for simulation of system. The released water from reservoir was calculated by continuous equation:

$$S_{t+1} = S_t + Q_t - R_t - e_t \Rightarrow R_t = S_t + Q_t - S_{t+1} - e_t \tag{32}$$

Where:

R_t: The volume of released water from reservoir in month t

Q_t: The volume of inflow to reservoir in month t

The simulation model improves results of optimization model. Therefore using of simulation model is necessary after optimization. In this research, simulation model shows the number of deficits in a period 30 years. For initial conditions, simulation model assumes that storage of reservoir is equal to determined optimum volume of reservoir by optimization model and the released water from reservoir is equal to water demand of downstream. Then simulation model finds S_{t+1} by trial error method. The results of simulation model are three types:

a) $S_{t+1} \rangle K_a \Rightarrow S_{t+1} = K_a \Rightarrow R = D$ In this situation, reservoir can supply water demand of the downstream of dam. (D: The volume of monthly water demand of the downstream of dam)

b) $S_{\min} \langle S_{t+1} \langle K_a \Rightarrow R = D$ In this situation, reservoir can supply water demand of the downstream of dam.

c) $S_{t+1} \langle S_{\min} \Rightarrow S_{t+1} = S_{\min} \Rightarrow R = 0$ In this situation, reservoir can not supply water demand of the downstream of dam.

RESULTS

The volume- area relation of the Amirkabir dam is shown by equation33.

$$A=0.6874+0.0167S \qquad (33)$$

Where:
A: The area of reservoir (Km^2)
V: The volume of reservoir (MCM)

The necessary data of yield model are shown in Tables1, 2 and3.

By using of the Markov chain method, 500 time series of volume of monthly inflow to reservoir were produced. These time series were 29 years. For evaluation of results different time series of produced data, 3 time series of produced data and main time serial were considered in this research. The necessary data and results of the Markov chain method are shown in Tables 4, 5 and 6.

Table 1: The volume of drinkable water demand and irrigation water demand

Month	January	February	March	April	May	June
Drinkable water demand(MCM)	18.3	18.4	19.4	19.55	22.07	26.2
Irrigation water demand(MCM)	5.57	4.82	3.13	5	21.8	26.5
Month	July	August	September	October	November	December
Drinkable water demand(MCM)	28.8	28.9	27.58	24.11	21.45	19.5
Irrigation water demand(MCM)	20.7	16.1	15.9	11.26	9	7.02

Table 2: The height of monthly evaporation in reservoir of the Amirkabir dam

Month	January	February	March	April	May	June
The height of monthly evaporation (m)	0.02	0.021	0.02	0.08	0.185	0.32
Month	July	August	September	October	November	December
The height of monthly evaporation (m)	0.422	0.41	0.336	0.275	0.17	0.112

Table 3: The values of β_t, $\wp_{ft}$ and $\wp_{pt}$.

Month	January	February	March	April	May	June
β_t	0.0399	0.043	0.0813	0.168	0.2045	0.1294
$\wp_{ft}$	0.1013	0.1045	0.0996	0.08	0.0735	0.0711
$\wp_{pt}$	0.0922	0.0702	0.0559	0.027	0.0571	0.0725
Month	July	August	September	October	November	December
β_t	0.0828	0.0614	0.052	0.042	0.0481	0.0471
$\wp_{ft}$	0.0695	0.0681	0.0704	0.082	0.087	0.0938
$\wp_{pt}$	0.0308	0.0188	0.0368	0.141	0.2469	0.1507

Table 4: The governing stochastic distributions on monthly inflow to reservoir

Month	January	February	March	April	May	June
The governing stochastic distributions	Log normal 3 Parameter	Log Pearson III	Log normal 3 Parameter	Pearson III	Log Pearson III	Normal
Month	July	August	September	October	November	December
The governing stochastic distributions	Log Pearson III	Log Pearson III	Normal	Log Pearson III	Pearson III	Log normal 3 Parameter

Table 5: The produced data of 3 time series and main data

Month	January	February	March	April	May	June
Mean volume of inflow to reservoir(MCM) time serial 1	12.3	13.9	25.2	73.5	101.3	76.9
Mean volume of inflow to reservoir(MCM) time serial 2	11.6	13.7	24.9	12.1	98	74.4
Mean volume of inflow to reservoir(MCM) time serial 3	12.7	14.3	25.3	14.8	102.4	76.8
Mean volume of inflow to reservoir(MCM) main time serial	12.6	14	24.8	74.3	101.9	76.6
Month	July	August	September	October	November	December
Mean volume of inflow to reservoir(MCM) time serial 1	41.7	19.7	12.7	11.4	13.7	14.5
Mean volume of inflow to reservoir(MCM) time serial 2	48.5	22.3	12.1	11	12.1	13.7
Mean volume of inflow to reservoir(MCM) time serial 3	42.8	20.1	12.6	11.6	14.8	15
Mean volume of inflow to reservoir(MCM) main time serial	41.7	19.6	12.6	11.6	14.5	14.9

Table 6: The yearly data of 3 produced time series by the Markov chain method and main data

Mean volume of yearly inflow to reservoir(MCM) time serial 1	417.7
Mean volume of yearly inflow to reservoir(MCM) time serial 2	354.3
Mean volume of yearly inflow to reservoir(MCM) time serial 3	363.2
Mean volume of yearly inflow to reservoir(MCM) main time serial	419

Table 7: The number of deficits of produced time series by the Markov chain method and main time serial for 4 scenarios

Time serial	Scenario	January	February	March	April	May	June
Time serial1	Scenario1	8	4	6	3	0	0
	Scenario2	5	6	6	2	0	0
	Scenario3	5	5	7	3	0	0
	Scenario4	5	5	7	1	0	0
Time serial2	Scenario1	9	13	10	0	0	0
	Scenario2	7	14	11	1	0	0
	Scenario3	3	18	10	1	0	0
	Scenario4	2	18	10	3	0	0
Time serial3	Scenario1	6	11	13	3	1	0
	Scenario2	6	9	13	5	0	1
	Scenario3	6	13	9	6	0	1
	Scenario4	5	11	11	5	0	1
Main time serial	Scenario1	7	4	6	1	0	0
	Scenario2	6	4	7	1	0	0
	Scenario3	4	6	4	2	0	0
	Scenario4	6	4	5	1	0	0
Time serial	Scenario	July	August	September	October	November	December
Time serial1	Scenario1	0	0	2	3	5	6
	Scenario2	0	0	2	3	6	5
	Scenario3	0	0	1	3	6	5
	Scenario4	0	0	2	1	8	4
Time serial2	Scenario1	0	1	3	4	9	16
	Scenario2	0	1	3	4	9	15
	Scenario3	0	1	2	5	8	17
	Scenario4	0	1	2	5	8	16
Time serial3	Scenario1	1	2	6	7	7	12
	Scenario2	1	1	6	6	10	10
	Scenario3	1	1	6	6	9	9
	Scenario4	1	1	6	6	8	11
Main time serial	Scenario1	0	0	2	3	4	3
	Scenario2	0	0	2	2	4	4
	Scenario3	0	0	1	3	3	5
	Scenario4	0	0	2	2	4	3

For optimization of volume of reservoir, four scenarios were considered. In these scenarios, drinkable water demand was satisfied perfectly. But irrigation water demand was prepared 0%, 66%, 68% and 70% respectively. For 4 scenarios, the calculated volume of reservoir by optimization model is 191.4,202,215 and 229 MCM respectively. Simulation model determines the number of deficits. The results of simulation model are shown in table7 for different time series and different scenarios.

CONCLUSION

The actual volume of reservoir of the Amirkabir dam is 191 MCM. This volume is almost equal to optimum volume of scenario1. This scenario supplies drinkable water demand only. In the other word this dam can not prepare irrigation water demand. Among prepared time series of data by the Markov chain method, the data of time series 2 and 3 show drought condition. The number of deficits of these scenarios is more than other scenarios and main time serial considerably. By increasing volume of reservoir and irrigation water demand, the number of deficits decreases. But this reduction is negligible. Simulation model shows that deficits often occur in winter. Inflows to reservoir of the Amirkabir dam are produced by snowmelt. Snowmelt occurs in spring therefore the number of deficits are very low in spring. After snowmelt, discharge of the Karaj River decreases and it reaches to its minimum value in winter. By attention to the number of deficits, the best season is spring for agricultural. In this season, irrigation water demand can be satisfied by reservoir and inflow to reservoir of the Amirkabir dam.

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