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On Sensitivity of the Chance Constrained Programming

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Abstract: Chance constrained programming is one of the common methods in programming under uncertainty but its results depend on selecting some important parameters used in the formulation. This paper considers the sensitivity of the chance constrained programming when different risk probabilities and methods are used. The results show that use of different risk parameters can results in very different solutions and conclusions and therefore different policy implications.

Key words: Chance constrained programming % Simulation % Sensitivity analysis

INTRODUCTION

In the real world many decisions should be made under uncertainty[1] because many important parameters that affect our decisions have unknown probability of occurrence. Many types of mathematical programming models have been developed for solving decision making problems under risky conditions. Generally, uncertainty can be included in each of (or all) three main parts of a mathematical programming model, i.e. objective function, technical coefficients and RHS (right hand side) [1]. One of the common mathematical programming methods that consider risk in RHS is chance constrained programming (CCP) [2-5]. Using this method risk can be included in the model both in technical coefficients and/or RHS parts. But there are some arbitrary parameters in such models that can affect final solutions and then policy implications. The main goal of this study is to analyze sensitivity of CCP models that consider risk in RHS when different parameters are selected. Also, for more reality some available data from capital resource in Iran's agricultural sector is used.

Theoretical Background: The chance-constrained formulation was introduced by Charnes and Cooper and deals with uncertain RHS's assuming the decision maker is willing to make a probabilistic statement about the frequency with which constraints need to be satisfied [6]. Namely, the probability of a constraint being satisfied is greater than or equal to a pre-specified value ": [7].

$$P\left(\sum_j a_{ij} x_j \le b_i\right) \ge a$$

Where a_{ij} is technical coefficient of j*th* decision variable in i*th* constraint, x_j is j*th* decision variable and b_i is i*th* RHS. By subtracting average value of the RHS (b_i) from both sides of the inequality and then dividing both sides by the standard deviation of the RHS (F_{bi}) then the constraint becomes:

$$P\left[\frac{\sum_j a_{ij} x_j - \bar{b}_i}{s_{b_i}} \le \frac{\left(b_i - \bar{b}_i\right)}{s_{b_i}}\right] \ge a$$

In the above formula the term $\frac{\left(b_i - \bar{b}_i\right)}{s_{b_i}}$ is number of standard errors (Z) that b_i is away from the mean. For a given probability limit (") there is an appropriate Z value as following:

$$P\left[\frac{\sum_j a_{ij} x_j - \bar{b}_i}{s_{b_i}} \le Z_a\right] \ge a$$

And then it can be rewritten as:

$$\sum_j a_{ij} x_j \le \bar{b} - Z_a s_{b_i}$$

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[1]Although risk and uncertainty are not really the same but they are used here interchangeably.

Based on this inequality total used resources must be less than or equal to average resource availability less the standard deviation times a critical value which arises from the probability level. Values of Z_α may be determined in two ways: [7] a) by making assumptions about the form of the probability distribution of b_i (for example, assuming normality and using values for the lower tail from a standard normal probability table); or b) by relying on the conservative estimates generated by using Chebyshev's inequality, which states the probability of an estimate falling greater than Z standard deviations away from the mean is less than or equal to one divided by Z^2. Using the Chebyshev inequality one needs to solve for that value of Z such that $(1-\alpha)$ equals $1/Z^2$. Thus, given a probability α, the Chebyshev value of Z_α is given by the equation $Z_\alpha = (1-\alpha)^{-0.5}$. Each method selected for calculating Z_α and based on any probability level α can results in different solutions. In this study such a sensitivity using simulation and Iran's agricultural sector data is analyzed.

SIMULATION AND RESUTLS

This section of the paper can be divided into two parts. In the first part, sensitivity of the Z values and then RHS values for different levels of probability level α and method selected for calculating Z_α is analyzed. In the second part, using average capital resource availability for Iran's farmer households in year 2006 the sensitivity of capital resource constraint is analyzed.

Sensitivity Analysis for the Z and RHS Values: The first factor that affects values of the Z and RHS in CCP models is probability level α or confidence level for meeting the resource constraint. The more probability level α or confidence level is selected, the more Z value and then the less RHS are created. In this study, a range of probability level α from 0.80 to 0.99 is selected. It means that we want to know what is the value of Z if probability of meeting the constraint is 80, 81, …, or 99 percent, Or equivalently, the probability of not meeting the constraint is 20, 19, …, or 1 percent, respectively. Table 1 shows the results of these calculations.

As table shows, increase in level of probability α results in decrease in Z value regardless of method of calculating it. But there are some more important findings. First, Z from Chebyshev's inequality is always greater than Z from standard normal probability table. Second, their difference becomes more as levels of probability α increases. For example, for levels of probability 80 percent Z from standard normal probability table and Chebyshev's inequality are 0.8416 and 2.2361 respectively and the ratio of Z from Chebyshev's inequality to Z from standard normal probability table (Z_{CS}) is 2.66. But at the end of range and for the highest confidence level (levels of probability 99 percent) Z values are 2.3263 and 10.000 respectively and the ratio is 4.30. Also, Growth rate values of these two types Z (ΔZ) confirm this finding. After probability level of 88 percent, growth rate of Z from Chebyshev's inequality becomes more and more than growth rate of Z from standard normal probability table. Figures 1 and 2 depict these findings separately for Z values and their growth rates. These findings show that selecting different probability level and different method of calculating Z_α may be result in very much different solutions and policy implications. For better testing this problem and using average capital resource availability for Iran's farmer households in year 2006 the sensitivity of capital resource constraint is analyzed and discussed in the next section.

Table 1: Z Values for different levels of probability α and method of calculating Z_α

	Z from standard normal probability table		Z from Chebyshev's inequality		
α	Δ Z (%)	Z	Δ Z (%)	Z	Z_{CS}*
0.80		0.8416		2.2361	2.66
0.81	4.3101	0.8779	2.5978	2.2942	2.61
0.82	4.2680	0.9154	2.7402	2.3570	2.57
0.83	4.2388	0.9542	2.8992	2.4254	2.54
0.84	4.2228	0.9945	3.0776	2.5000	2.51
0.85	4.2209	1.0364	3.2796	2.5820	2.49
0.86	4.2343	1.0803	3.5098	2.6726	2.47
0.87	4.2646	1.1264	3.7749	2.7735	2.46
0.88	4.3143	1.1750	4.0833	2.8868	2.46
0.89	4.3865	1.2265	4.4466	3.0151	2.46
0.90	4.4861	1.2816	4.8809	3.1623	2.47
0.91	4.6197	1.3408	5.4093	3.3333	2.49
0.92	4.7970	1.4051	6.0660	3.5355	2.52
0.93	5.0332	1.4758	6.9045	3.7796	2.56
0.94	5.3519	1.5548	8.0123	4.0825	2.63
0.95	5.7938	1.6449	9.5445	4.4721	2.72
0.96	6.4342	1.7507	11.8034	5.0000	2.86
0.97	7.4318	1.8808	15.4701	5.7735	3.07
0.98	9.1959	2.0537	22.4745	7.0711	3.44
0.99	13.2732	2.3263	41.4214	10.0000	4.30

* Ratio of Z from Chebyshev's inequality to Z from standard normal probability table

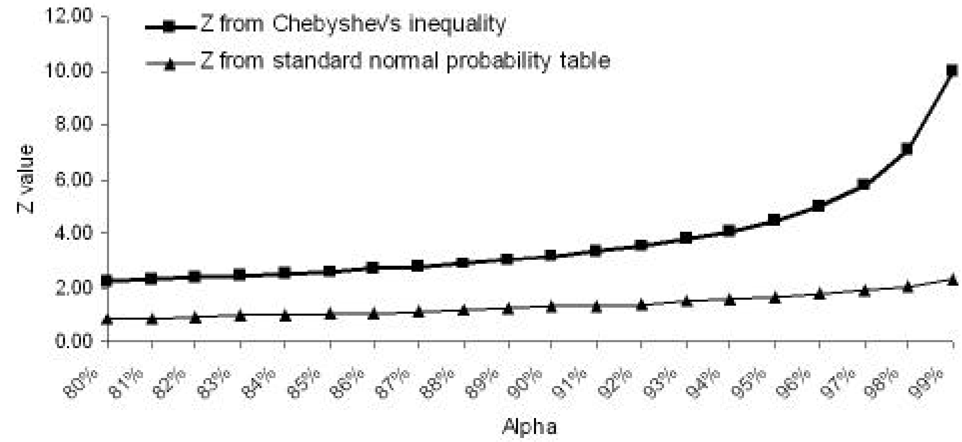

Fig. 1: Z Values for different levels of probability "and method of calculating Z"

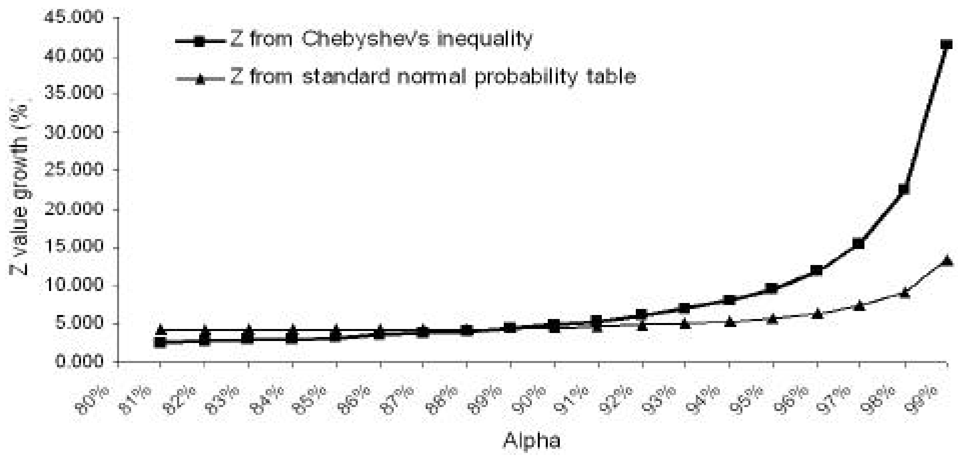

Fig. 2: Growth rate of Z for different levels of probability " and method of calculating Z"

Table 2: Results of sensitivity analysis for capital resource constraint

"	RHS with Z from standard normal probability table	RHS with Z from Chebyshev's inequality
0.80	31321	19755
0.81	31020	19273
0.82	30709	18752
0.83	30387	18185
0.84	30053	17566
0.85	29705	16886
0.86	29341	16134
0.87	28959	15297
0.88	28556	14358
0.89	28128	13293
0.90	27672	12073
0.91	27181	10654
0.92	26647	8977
0.93	26061	6952
0.94	25406	4441
0.95	24659	1209
0.96	23781	-3169
0.97	22702	-9585
0.98	21267	-20347
0.99	19006	-44640

Sensitivity Analysis for Capital Resource Constraint: As mentioned above, most of decisions are involved in different levels of uncertainty. One of the most important factors needed for any business is cash capital. Value of such a resource in some activities like farming is hardly predictable. In other words, capital resource availability is often involved in high level of uncertainty. In this study, capital resource constraint for a typical Iranian farmer household is used for testing and analyzing the CCP sensitivity as explained above. Based on the available data, the value of capital resource available for an Iranian farmer household in year 2006 was about 38 million Rials on average and with 8300 Rials standard deviation [8]. In other words, value of RHS for capital resource constraint in a typical Iranian farm model in year 2006 was 38 million Rials. Using these data and for different levels of probability " and method of calculating Z" this RHS can be modified for including uncertainty in the model. The results for such modification are shown in table 2. As results show, value of RHS in capital resource constraint becomes less and less as levels of probability " increase but this decrease when using Z from Chebyshev's inequality is very much more than Z from standard normal probability table. Furthermore, if confidence level is more than 95 percent the value of RHS when using Z from Chebyshev's inequality becomes negative! As mentioned above, this is because of characteristics of somewhat "exaggerated" calculation of Z value based on Chebyshev's inequality.

CONCLUSIONS

In the real world many decisions should be made under uncertainty because many important parameters that affect our decisions have unknown probability of occurrence. Many types of mathematical programming models have been developed for solving decision making problems under risky conditions. Chance constrained programming is one of the common methods in programming under uncertainty but its results depend on selecting some important parameters used in the formulation. This paper considers the sensitivity of the chance constrained programming when different risk probabilities and methods are used. The results show that use of different risk parameters can be result in very different solutions and conclusions and therefore different policy implications. The result of applying this analysis for real data from Iran's agricultural sector shows that one must greatly pay attention to this sensitivity. In fact, there is a trade-off between increasing probability of meeting a constraint and the model flexibility and any one must consider such sensitivity when modeling risk using chance constrained programming.

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