American-Eurasian J. Agric. & Environ. Sci., 18 (2): 64-76, 2018
ISSN 1818-6769

DOI: 10.5829/idosi.aejaes.2018.64.76

Comparison of Methods for Fitting the Theoretical Variogram to the Experimental Variogram for Estimation of Depth to Groundwater and its Temporal and Spatial Variations

Hamid Mohebzadeh

Department of Water Engineering, Faculty of Agriculture, Bu-Ali Sina University, Hamedan, Iran

Abstract: Detecting of variations of depth to groundwater, temporarily and spatially, is a main factor to obtain sustainable water use in the basin. Usually point measurements of depth to groundwater are available in any area, but what is needed is groundwater surfaces based on these measurements that show condition of depth to groundwater. For simulation of this surface a robust interpolation method is needed. The main purpose of this study was to compare two different fitting methods, ordinary least squares (OLS) and weighted least squares (WLS), for fitting theoretical variogram models to the experimental variogram based on six selected years (1982, 1991, 1996, 2001, 2006, 2012) as starting points. The cross-validation method was utilized to evaluate the precision of the methods and two indicators – the correlation coefficient (R^2) and the root mean squared error (RMSE) were used to compare the different variogram models fitting by two fitting methods. Two other indexes – deviation of estimation errors (σ) and 95% prediction interval (95 PPI) were used for evaluation of prediction errors. Results showed that the circular model fitted by WLS method with respectively lowest and highest values of RMSE and R^2 is the optimal method for interpolating depth to groundwater. It also contained the minimum standard deviation of estimation errors and lowest 95% prediction interval (95 PPI). Spatial distribution maps of depths to groundwater in different years indicated that although the depths to groundwater in this region are relatively deep but small fluctuations of depth to groundwater have happened in the past 30 years. Temporal variation of depth to groundwater showed that the water table has risen over the past 30 years, with average of 0.22 m/yr.

Key words: Depth To Groundwater · Spatial And Temporal Variation · Kriging · Geor Package

INTRODUCTION

Groundwater simulation models today play an active role in the development and utilization of logical water policies. Since the validity of the simulation depends highly on the existing data, the duty of effective use of the observation networks is very importance [1]. Now most researches are focused on the exchange of surface water and groundwater and groundwater system model. Since the cost of the installation and maintenance of a groundwater monitoring network is extremely high, an optimal monitoring network should be designed, which leads to the research of proper network design method [2]. Depth to groundwater measurements are accessible whereas spatial distribution of groundwater surfaces based on these measurements are required. To do this an exact interpolation method is required and a great number of studies have discussed in the literature. Among different methods of interpolation, no method is uniquely desirable and therefore the most suitable interpolation method for a particular situation can be achieved only by comparing their results. Deterministic and Geostatistical techniques are two main interpolation methods. The deterministic method is utilized for making surfaces from point measurements and rely on the level of similarity (e.g., IDW). The geostatistical interpolation approach is based on statistics and comprises errors or uncertainty of predictions and can be applicable for more progressive prediction surface modeling.

Between various interpolation techniques, the inverse distance weighting (IDW) method is the most popular method for estimation of missing data in

Corresponding Author: Hamid Mohebzadeh, Department of Water Engineering, Faculty of Agriculture, Bu-Ali Sina University, Hamedan, Iran. Tel: +98 9193209187, Fax: +98-21-55225413.

hydrology and geosciences [3]. Several variants of IDW are derived and adopted by researchers with a main focus on the weighting schemes. In fact the prosperity of the IDW method relies fundamentally on the existence of positive spatial autocorrelation [4], since data from places close to each other in space are more likely to be the same than the data from locations away from one another [5]. Unfortunately, this condition is not always true and then inserts arbitrariness in the choice of weighting parameters. Deterministic methods such as IDW also have weaknesses. For example, in IDW there is no specific way for specify the power and neighborhood radius. Thus was tried to develop a linear estimator that act based on average weighted of observations and has specific performance for determination of the weights that led to invention of kriging method by Krige [6].

Various studies have been accomplished to explain the spatial and temporal distribution of groundwater level fluctuations. The most advantageous tool for analyzing such processes is geostatistical techniques. Spatial analysis of groundwater level of 31 wells were carried out by Theodossiou and Latinopoulos [1] employing kriging, but no temporal analysis was done. However, they provide some statistical analysis like maximum and minimum value, mean, median and standard deviation based on their data from two years of observation. Olea and Davis [7] in an extensive study used the kriging and cross validation method for estimation of the water level at each observation well. In addition, some new observation wells were proposed based on the standard deviation of kriging. Prakash and Singh [8] used the technique of kriging and specified the appropriate number of observation wells that can be added to the available network to monitor the spatial distribution of groundwater level. Ahmadi and Sedghamiz [9] utilized the kriging and cokriging methods to check their accuracy in groundwater depth mapping. Their results showed that cokriging provide more accurate results in mapping the groundwater depth across the study area. Overally, results depicted that kriging and cokriging represent uncertainty in estimations, since they underestimated the groundwater depth for dry, wet and normal conditions.

Between the studies focusing on temporal analysis of groundwater, Kumar and Ahmed [10] evaluated the groundwater level within 12 months of the year and applied the kriging technique for estimation of groundwater level in some unmeasured points and wells for each month. Sun, Kang [11] studied the groundwater level of the Minqin oasis located in northwest China utilizing a kriging method and calculate the precision of results using cross-validation method. Subsequently, the most suitable interpolation method was applied for spatio-temporal analysis of depth to groundwater. Li and Revesz [12] used various interpolation methods to examine spatio-temporal variations of regionalized variables and improved their existing dataset with some measurements in different time periods. Machiwal, Mishra [13] integrated geostatistics and GIS in order to model spatio-temporal variations of groundwater levels to discover the behavior of the hard-rock aquifer systems. Experimental variogram of the observed groundwater levels was calculated using 750-m lag distance interval and the four most frequently used geostatistical methods were fitted to the experimental variogram. Groundwater maps revealed that the groundwater levels are forcefully influenced by surface topography and the existence of surface water bodies in the study area. Also, temporal variation of the groundwater levels was significantly regulated by the wet-season recharge and amount of groundwater exploitation.

The main objectives of this study were firstly to select an optimal method for fitting theoretical variogram models namely, spherical, circular, Gaussian and exponential, to the experimental variogram in the study area, among fitting methods (including ordinary least squares (OLS) and weighted least squares (WLS)). We did so by comparing the accuracy of each fitting method and analyzing the errors. Secondly, the temporal and spatial variations of depth to groundwater in the study area were interpolated by performing the ordinary kriging using the parameters (sill, range and nugget) estimated by the optimal fitting method. All interpolations and geostatistical analysis were carried out using the statistical software GNU R, version 2.15.2 and the add-on package geoR [14].

MATERIALS AND METHODS

Study Area and Data Collection: This study was carried out on the plain in central of Nebraska state, which occupies an area of 1428 km^2 (longitude from 97°20'W to 97°50'W and latitude from 40°40'N to 41°05'N). Nebraska is one of the leading agricultural states in the USA with over 3.4 million ha of irrigated land and about 5 million ha of natural grassland. The average annual precipitation (January 1–December 31) is about 550 mm. The long-term statewide average growing season (May 1–September 30) rainfall is about 360 mm. The maximum long-term average growing season precipitation occurs in the southeast part of the state and the minimum occurs on the western edge.

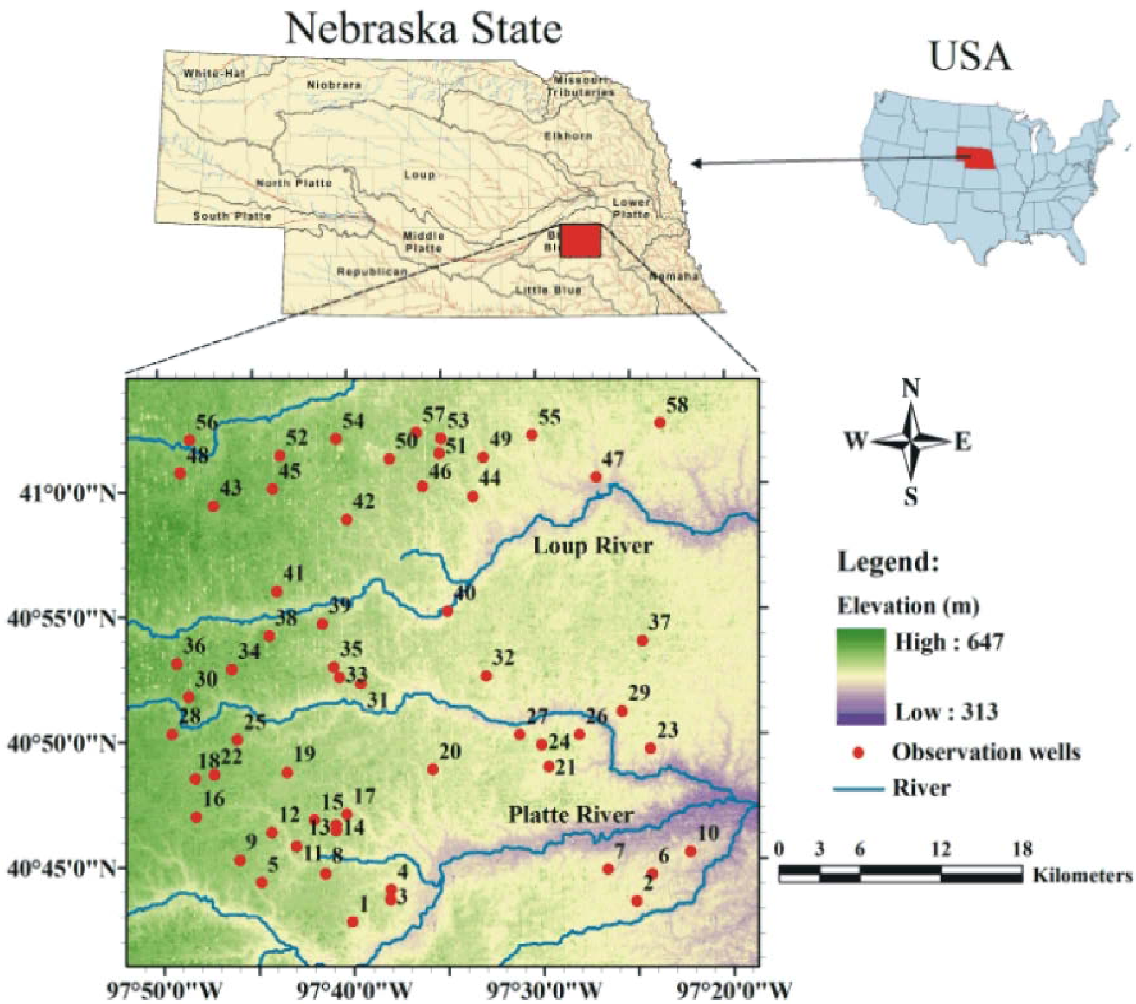

Fig. 1: Location of the study area and position of the observation wells in Nebraska State

While the greater part of precipitation occurs in spring, the distribution in the calendar year shows significant fluctuations. Nebraska has a continental climate, with highly variable temperatures from season to season and year to year. The central region has an average annual normal temperature of 10°C, with a normal monthly maximum of 24°C in July and a normal monthly minimum of –6°C in January. The prevailing cropping pattern in the area is field maize-soybean rotation and a considerable area of croplands in the region are irrigated with center pivots with extracted water from the Ogallala aquifer that is the main source of irrigation water [15].

The monthly average observed depth to groundwater dataset used in this study has been provided from USGS (U.S. Geological Survey) and comes from 58 observation wells over the period of 1982-2012. Study area and observation wells are shown in Fig. 1. Based on the average monthly depth, a dataset of mean annual depth created for each observation well. Within the 30-year dataset, depth to groundwater in 1982, 1991, 1996, 2001, 2006 and 2012 was selected as reference points comparing various fitting methods and analysis spatial and temporal variability of depth to groundwater over the period 1982-2012.

Statistical Analysis of Dataset: Basically, the hypothesis of kriging is that the target variable is stationary and has a normal distribution that is probably the largest restriction of kriging. The histogram displays the frequency distribution for the dataset. Therefore, normality of the spatial depth to groundwater dataset was checked before geostatistical modeling by plotting histograms and by applying one of the most powerful statistical tests, i.e., Shapiro–Wilk (SW) test, for six selected years, during the period of 1982-2012 using R software. The Shapiro–Wilk test uses the null hypothesis principle to determine whether a sample y_1, ..., y_n is normally distributed from a population. This test was the first test that was capable to indicate departures from normality owing to either skewness or kurtosis, or both [16]. The Shapiro–Wilk test statistic is defined as:

$$W = \frac{\left(\sum_{i=1}^{n} a_i y_i\right)^2}{\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2} \quad (1)$$

where y_i is the i^{th} order statistic, $\bar{y}$ is the sample mean,

$$a_i = \left(a_1, \ldots, a_n\right) = \frac{m^T v^{-1}}{\left(m^T v^{-1} v^{-1} m\right)^{1/2}}$$

and $m=(m_1, \ldots, m_n)^T$ are the expected values of the order statistics of independent and identically distributed random variables sampled from the standard normal distribution and is the covariance matrix of those order statistics.

The value of W is between zero and one. The small value of W result in rejection of normality, while the value of one represents normality of the data.

The function to carry out this test has an element that is called p-value. If the value of p is greater than 0.05, the data is normal. If this value is less than 0.05, then the data is significantly apart from a normal distribution.

Ordinary Kriging Method and Experimental Variogram: There are various interpolation methods, e.g., kriging, inverse distance weighting, deterministic splines, etc. However, kriging, or best linear unbiased estimation (BLUE), given only the variogram, has been the most used in mining, geology and hydrology [17]. A significant superiority of kriging is that it is more flexible compared with other interpolation methods [17]. The weights are not choose based on some optional principal that may be suitable in some cases but not in others, but depend on how the function varies in space. Data can be interpreted in a systematic and objective way and previous experiences are used to create a variogram, which then is used to specify the adequate weights [13].

The geostatistical methods such as kriging, use a specific function which is called experimental variogram, in order to calculate the difference between observations and to evaluate the weights λ_i (Eq. 2).

$$\hat{Z}(x_0) = \sum_{i=1}^{N} \lambda_i Z(x_i) \quad (2)$$

where $\hat{Z}(x_0)$ = estimation of function $Z(x)$ at point x_0 and λ_i = weighting factors.

The first step in kriging is to calculate an experimental variogram, computed as half of the mean squared of the components of data pairs, using the equation below:

$$\hat{\gamma}(h) = \frac{1}{2m(h)} \sum_{i=1}^{m(h)} \left[z(x_i) - z(x_i + h)\right]^2 \quad (3)$$

where $\hat{\gamma}(h)$ is the estimated semi-variance for the distance h and $m(h)$ is the number of calculated point pairs in the vector distance class h. The experimental variogram is a function between distance and direction and when the field has isotropy, it is relatively simple to compute, which depends only on h and independent from its direction. In the absence of isotropy (anisotropic spatial patterns), various variograms should be extracted in the typical groups of directions.

Methods for Fitting Theoretical Variogram to the Experimental Variogram: Experimental variogram estimators are useful tools for exploring spatial correlation, but they can not be used in spatial regression or prediction, because they do not necessarily imply a valid spatial process. Hence, the experimental variogram curve is defined by another theoretical curve with a determined mathematical formula. This smooth curve fitted to the experimental variogram is called theoretical variogram. The most generally used theoretical variogram models are spherical, circular, Gaussian and exponential. Choosing and matching a theoretical variogram is something of an art , not science [18]. Many packages supply a default model and try to find the best set of parameters (sill, nugget and range) to fit the dataset, whereas other models utilize this approach for all models, which they support and choose the one with the highest correlation coefficient or lowest sum of squares residuals. In this study two methods including: ordinary least squares (OLS) and weighted least squares (WLS) were used for fitting theoretical variogram model to the experimental variogram, which the details of these methods are described below.

Ordinary and Weighted Least Squares: Ordinary least squares (OLS) regression is a generalized linear modeling technique that can be utilized to model a single response variable that is recorded in at least one interval scale. Different from linear and nonlinear least squares regression, weighted least squares (WLS) regression is not connected with a specific type of function used to describe the relationship between the process variables. Instead, weighted least squares reflect the behavior of the random errors in the model; and it can be used with functions that are either linear or nonlinear in the parameters [19]. Suppose we have estimated the experimental variogram $\gamma_z^*(h)$ at MAXLAGS=k distance classes, where $\gamma_z^*(h)$ can be either the classical estimate $\hat{\gamma}_z(h)$ or the robust estimate $\bar{\gamma}_z(h)$. In fitting based on least squares, we want to estimate the parameters vector θ of the theoretical variogram $\gamma_z(h)$ that minimizes the sum of square differences $R(\theta)$ given by the following expression:

$$R(\theta) = \sum_{i=1}^{k} w_i^2 \left[\gamma_z^*(h_i) - \gamma_z(h_i;\theta) \right]^2 \tag{4}$$

For i=1, …, k, the weights are $w_i^2 = 1/Var\left[\gamma_z^*(h_i)\right]$ in the case of WLS and $w_i^2 = 1$ in the case of OLS. Hence, the parameter θ is estimated in OLS by minimizing the expression below:

$$R(\theta)_{OLS} = \sum_{i=1}^{k} \left[\gamma_z^*(h_i) - \gamma_z(h_i;\theta) \right]^2 \tag{5}$$

For WLS, Cressie [20] examined estimations for the variance of both the classical and robust experimental variogram. Then, under the assumptions of normally distributed observations and uncorrelated squared differences in the experimental variogram, the approximate weighted least squares estimate of the parameters can be obtained by minimizing the following expression:

$$R(\theta)_{WLS} = \frac{1}{2} \sum_{i=1}^{k} N(h_i) \left[\frac{\gamma_z^*(h_i)}{\gamma_z(h_i;\theta)} - 1 \right]^2 \tag{6}$$

where $N(h_i)$ is the number of pairs of points in the i^{th} distance lag.

The main preference that valued weighted least squares over other methods is the capability to handle regression situations in which the data points are of different quality. The greatest drawback of weighted least squares is likely the fact that the theory of this method is based on the hypothesis that the weights are known exactly. This is almost never the case in real applications, of course, so estimated weights must be used instead [21].

Cross-Validation: The cross-validation method is applied to specify the method that gives the best result. In a cross-validation exercise, the estimation method is evaluated at the locations of available measurements. The sample value at a specific location is temporarily removed from the sample dataset and then, using the remaining samples, the value at the same location is estimated. When the estimate is computed, it can be compared to the actual sample value that was firstly discarded from the sample dataset. This procedure is repeated for all available samples. The resulting true and estimated values can then be compared using statistics [22]. In this way, we calculated the error between the actual and the estimated value so that the accuracy of each interpolation method was calculated.

The main selected criterion of cross-validation is root mean squared error (RMSE), which can be calculated as follows:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(z_i - \hat{z}_i \right)^2} \tag{7}$$

where $\hat{z}_i$ is the estimated value; z_i is the measured value at sampling point i (i=1, …, n); n is the number of values used for the estimation.

Another criterion to assess the fitting method is the correlation coefficient (R^2), which can summarize the correlation between the observed and estimated values. High R^2 indicates the strong correlation between observed and estimated variables.

Analysis of Uncertainties in Prediction: In the interpolated prediction, factors like the number of adjacent samples, the vicinity of the existing samples, the spatial adjustment of the samples (clustering) and the characteristics of the phenomenon, which is under study will affect the results. The standard deviation of the estimation error ( ) is an uncertainty index based on the number of adjacent data points, the vicinity of the samples, as well as the interaction between the different factors and is defined as:

$$\sigma = \sqrt{\sigma^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j C_{ij} - 2 \sum_{i=1}^{n} \lambda_i C_{oi}} \tag{8}$$

where σ^2 shows the variance of point values. The second term in the square root is a weighted sum of all the covariance between the pair of samples and computes the degree of clustering. The third term is a weighted sum of the covariance among the sample points and the estimated value and it measures the vicinity to each other of the available samples. The estimation is reliable when it treats on a very smooth and well-behaved variable and the samples are not very close together but close to the estimated samples [11].

RESULTS AND DISCUSSION

Results of Statistical Analysis of Dataset: Fig. 2 shows the histograms of the depths to groundwater for six selected years. It is obvious that the depths to groundwater follow an approximately normal distribution in all the years. Moreover, the results obtained from Shapiro–Wilk (SW) test confirm normality of the depth to groundwater. Observed SW test-statistics indicate that null hypothesis of the presence of normality in the depths to groundwater cannot be rejected at 1% significance level as p-value is greater than 0.05 for all the years except year 2006 (Fig. 2a–f). Therefore, it can be concluded that the dataset of depths to groundwater come from a normally distributed population and hence, are appropriate for geostatistical analysis.

Comparison of Fitting Methods and Analysis of Uncertainties in Prediction: Table 1 shows parameters of the four theoretical variogram models fitted to the yearly depth of groundwater of six selected years by using OLS and WLS fitting methods. Also, the correlation coefficient (R^2) and root mean squared error (RMSE) between the estimated and observed depth to groundwater are represented in Table 2. It can be seen from Table 2 that two theoretical variogram models, circular and spherical, are the best-fit models with maximal correlation coefficient and minimal RMSE. However, according to table 2 it is obvious that the circular model fitted by WLS method is the optimal method for interpolating depth to groundwater in the study area. The best result of circular model variogram was obtained for year 1991 that is shown in Fig. 3 as an example. The scatter plots of the predicted values of depth to groundwater, obtained by circular model fitted by WLS method, versus observed values of six selected years are shown in Fig. 4. According to Fig. 4 the R^2 values range between 0.74 (year 1996) and 0.81 (year 1991). The high values of R^2 verify that the circular model fitted by WLS technique is the most accurate model, which can be used by kriging mothed for analysis of the spatio-temporalvvariations of depth to groundwater in the study region. The precision of kriging values relies significantly on the variogram values at small lags [23].Generally, the nugget-to-sill proportion can be utilized to classify the spatial dependency [24]. If the ratio is less than 0.25, a variable is assumed to have strong spatial dependence and has a mild spatial dependence if the ratio lies between 0.25 and 0.75; otherwise, the variable has a weak spatial dependence [24]. In this study the value less than 0.25 was detected for all the years. Therefore, depths to groundwater have strong spatial dependence in the study area.

In this study, Eq. (8) was used to analyze the uncertainties of the theoretical variogram models and 95% prediction intervals were calculated. As shown in Table 3, the standard deviation of estimation error for several theoretical variogram models ranged from 5.17 to 7.19 and the circular model fitted by WLS method had the lowest standard deviation of estimation error and confidence interval, indicating that this method had the least uncertainties and the highest confidence intervals.

Spatial Variation of Depth to Groundwater: Ordinary kriging technique and circular theoretical variogram model fitted by WLS method were used for interpolation of the temporal and spatial variations of depth to groundwater in the study area; henceforth the spatial and temporal distribution of depths to groundwater in the region were obtained.

Fig. 5 (a-f) shows the spatial variation of depth to groundwater obtained by the ordinary kriging method in the region. We found that the depth to groundwater generally increased from the northwestern parts of the region to its southeastern parts. The deepest groundwater level was located in a zone in the southeastern part. It is obvious from Fig. 5 (a-f) and Fig. 6 that the depths to groundwater in this region are relatively deep (within a depth range of 50–115 m). In 1982, the area with depth to groundwater below 70m was limited to about 182 km^2 in the northwestern and northeastern part of the region; the main part of the area, with depth to groundwater of 80–90 m, have an area of 569 km^2; other deep areas, with depth to groundwater of 100–110 m, occurred in southeastern part with area of 57.52 km^2 (Fig. 5a). In 1991, however, the area with depth to groundwater below 70 m had declined to 155 km^2, also the area with depth to groundwater of 80–90 m had declined to 540 km^2; on the other hand, the area with depth to groundwater of 100–110 m had increased to 79.2 km^2 (Fig. 5b). The inverse trend was observed during the period of 1991–2001; the area with depth to groundwater below 70 m had increased to

Table 1: Parameters of four theoretical variogram models fitted by OLS and WLS methods for depth to groundwater of six selected years.

		OLS				WLS			
Year	Model Parameters	Spherical	Exponential	Gaussian	Circular	Spherical	Exponential	Gaussian	Circular
1982	Nugget (m^2)	0	0	13	0	0	0	9	0
	Sill (m^2)	184	318	172	180	173	204	167	174
	Range (m)	30000	31000	13900	25900	26000	15000	13000	24000
1991	Nugget (m^2)	0	0	15	0	0	0	8	0
	Sill (m^2)	185	312	173	181	175	206	171	176
	Range (m)	30000	30000	14000	25000	26700	15000	13000	24000
1996	Nugget (m^2)	4	2	24	5	0	0	11	0
	Sill (m^2)	217	331	204	213	213	243	205	213
	Range (m)	29000	25000	15000	26000	26000	14000	13000	24000
2001	Nugget (m^2)	0	0	15	0	0	0	6	0
	Sill (m^2)	253	634	202	208	195	286	196	193
	Range (m)	45000	72000	17000	30000	30000	25000	15000	26000
2006	Nugget (m^2)	0	0	12	0	0	0	7	0
	Sill (m^2)	147	214	137	147	141	154	135	142
	Range (m)	27000	22000	13000	24000	24000	12000	12000	22000
2012	Nugget (m^2)	0	0	14	0	0	0	7	0
	Sill (m^2)	255	693	228	234	225	307	225	224
	Range (m)	38000	67000	15000	29000	30000	22000	15000	26000

Table 2: Comparison of correlation coefficient and RMSE between observed values of depth to groundwater and simulated values by four theoretical variogram models fitted by OLS and WLS methods

		1982		1991		1996		2001		2006		2012	
Method		RMSE (m)	R^2	RMSE (m)	R^2	RMSE (m)	R^2	RMSE (m)	R^2	RMSE (m)	R^2	RMSE (m)	R^2
OLS	Spherical	5.49	0.78	5.37	0.79	6.69	0.74	5.80	0.77	5.51	0.73	6.22	0.77
	Exponential	5.50	0.78	5.39	0.79	6.84	0.72	5.80	0.77	5.55	0.73	6.26	0.77
	Gaussian	6.27	0.71	5.97	0.74	7.12	0.70	6.09	0.74	5.70	0.70	6.54	0.75
	Circular	5.41	0.78	5.19	0.80	6.51	0.75	5.80	0.77	5.34	0.75	6.54	0.75
WLS	Spherical	5.45	0.78	5.27	0.80	6.69	0.74	5.70	0.77	5.43	0.74	6.16	0.77
	Exponential	5.48	0.79	5.38	0.79	6.85	0.72	5.80	0.77	5.51	0.73	6.23	0.77
	Gaussian	6.32	0.71	6.05	0.74	7.12	0.70	6.02	0.75	5.80	0.70	6.57	0.75
	Circular	5.35	0.79	5.12	0.81	6.61	0.74	5.70	0.78	5.15	0.77	6.20	0.77

Table 3: Parameters of uncertainty analysis for theoretical variogram models

		1982		1991		1996		2001		2006		2012	
Method		σ(m)	95 PPI*	σ(m)	95 PPI	σ(m)	95 PPI	σ(m)	95 PPI	σ(m)	95 PPI	σ(m)	95 PPI
OLS	Spherical	5.53	$Z^*\pm1.42$	5.42	$Z^*\pm1.40$	6.74	$Z^*\pm1.73$	5.86	$Z^*\pm1.50$	5.55	$Z^*\pm1.43$	6.28	$Z^*\pm1.62$
	Exponential	5.55	$Z^*\pm1.43$	5.44	$Z^*\pm1.40$	6.90	$Z^*\pm1.78$	5.87	$Z^*\pm1.51$	5.60	$Z^*\pm1.44$	6.31	$Z^*\pm1.62$
	Gaussian	6.31	$Z^*\pm1.63$	6.02	$Z^*\pm1.55$	7.19	$Z^*\pm1.85$	6.15	$Z^*\pm1.58$	5.85	$Z^*\pm1.50$	6.60	$Z^*\pm1.70$
	Circular	6.32	$Z^*\pm1.62$	5.24	$Z^*\pm1.35$	6.57	$Z^*\pm1.69$	5.87	$Z^*\pm1.51$	5.39	$Z^*\pm1.39$	6.50	$Z^*\pm1.70$
WLS	Spherical	5.50	$Z^*\pm1.41$	5.32	$Z^*\pm1.37$	6.75	$Z^*\pm1.74$	5.78	$Z^*\pm1.49$	5.48	$Z^*\pm1.41$	6.22	$Z^*\pm1.60$
	Exponential	5.53	$Z^*\pm1.42$	5.43	$Z^*\pm1.38$	6.90	$Z^*\pm1.78$	5.84	$Z^*\pm1.50$	5.58	$Z^*\pm1.43$	6.29	$Z^*\pm1.69$
	Gaussian	6.37	$Z^*\pm1.64$	6.10	Z*±1.57	7.18	$Z^*\pm1.85$	6.08	$Z^*\pm1.56$	5.90	$Z^*\pm1.52$	6.64	$Z^*\pm1.70$
	Circular	5.40	$Z^*\pm1.39$	5.17	Z*±1.33	6.67	$Z^*\pm1.71$	5.74	$Z^*\pm1.48$	5.20	$Z^*\pm1.34$	6.26	$Z^*\pm1.61$

* PPI= 95 % prediction interval (m)

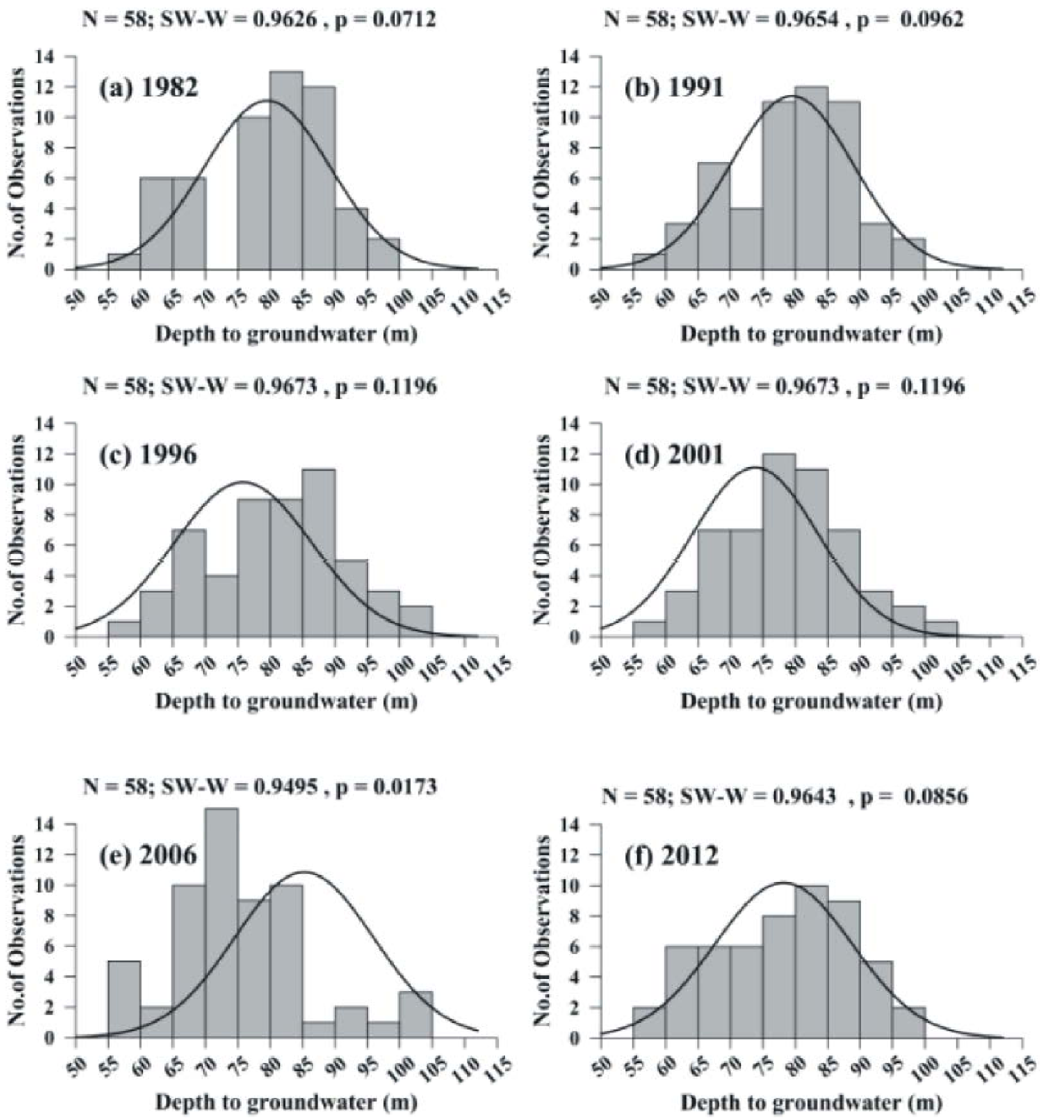

Fig. 2: (a-f) Histograms and Shapiro–Wilk test statistics of yearly depth to groundwater for six selected years

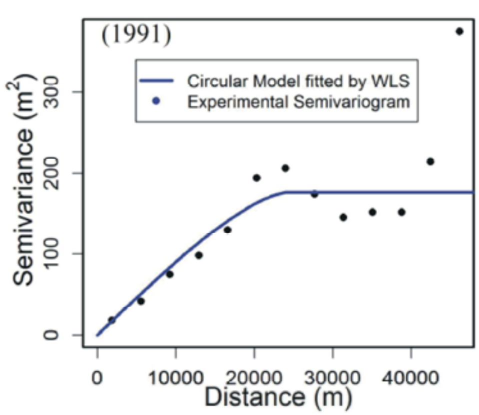

Fig. 3: Circular variogram model fitted by WLS method to the experimental variogram for depths to groundwater of year 1991

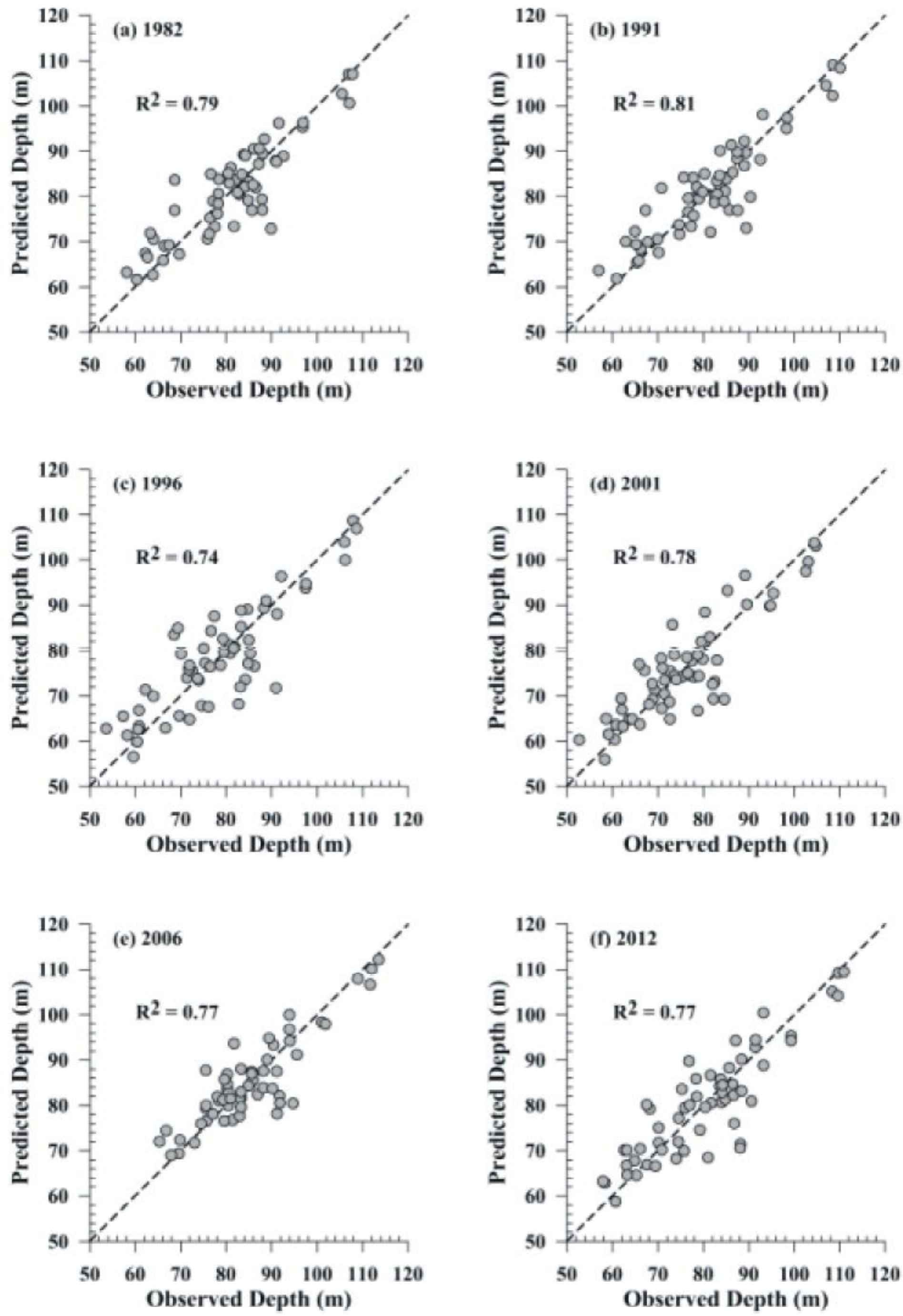

Fig. 4: Scatter plots between the observed and predicted depths to groundwater (by circular model fitted by WLS method) for six selected years

394 km^2, while that the area with depth to groundwater of 80–90 m and 100-110 had dwindled to 265.28 and 41.24 km^2, respectively. By 2012, the area with depth to groundwater less than 70 m had dwindled again and amounted to only 271 km^2; the area of 80–90 m depth had increased to 443 km^2. Moreover, the area with depth to groundwater greater than 100 m grew to 98 km^2 (Fig. 5f). It is apparent from Fig. 5, although spatially the depth to groundwater in the region is relatively deep, but during the period of 1982-2012 no significant variation can be

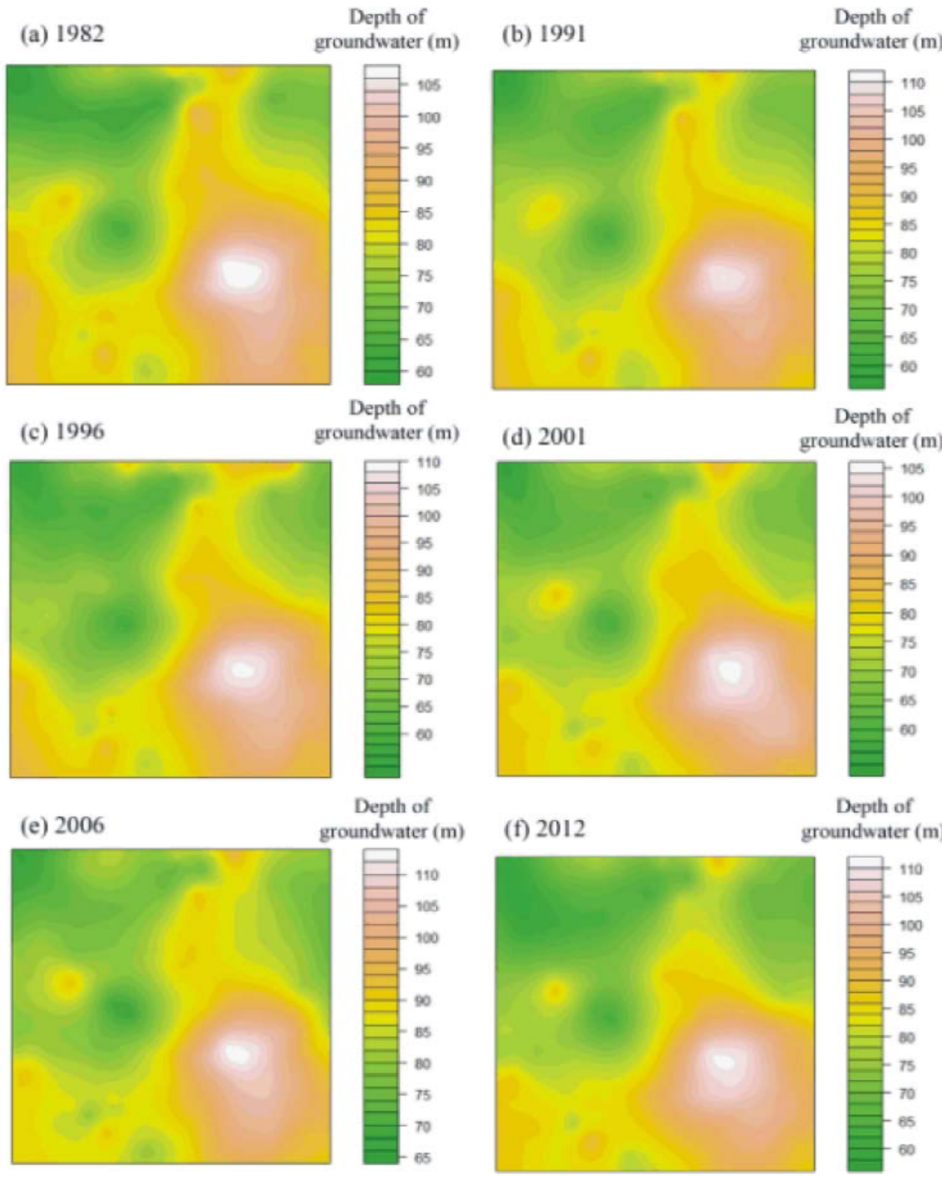

Fig. 5: Spatial distribution of depth to groundwater in the study area

observed in depth to groundwater. As mentioned earlier, because the region is limited in surface water resources, groundwater is the major source of irrigation water. Excessive groundwater extraction is the main cause of the decline in the water table, but since most of the croplands in the region are irrigated with center pivots, this procedure resulting in the small fluctuations of depth to groundwater in the past 30 years. However, water management activities, e.g., changing cropping pattern and growing less water requiring crops, are crucial for entire of the region and especially for the central and southeastern parts of the area.

Temporal Variation of Depth to Groundwater: As shown in Table 4, however spatially the depth to groundwater in the study area is mainly variable, the value of minimum, maximum and average depth to groundwater in six selected years were not significantly different. Over the past 30 years, the water table has not faced with progressive drop. According to table 4, during the period of 1982-2001 water table had raised from depth of 88.90 m to 75.72 m with rate of 0.7 m/yr. But during the period of 2001-2006 water table had declined about 10m with decline rate of 1.88 m/yr. Finally, during the period of 2006-2012, water table had raised again about 5m with rate

Table 4: Statistical results of depth to groundwater for 58 observation wells

Year	Mean depth (m)	Minimum depth (m)	Median depth (m)	Maximum depth (m)	Decline rate (m/yr)	Changing trend	Mean decline rate (m/yr)
1982	88.90	65.89	88.54	113.15			
1991	83.09	57.02	83.05	112.92	-0.64	↑	-0.22↑
1996	78.39	53.60	77.40	108.68	-0.94	↑	
2001	75.72	52.67	73.80	104.71	-0.53	↑	
2006	85.15	65.34	83.31	113.52	1.88	↑	
2012	80.02	57.90	80.38	110.96	-0.85	↑	

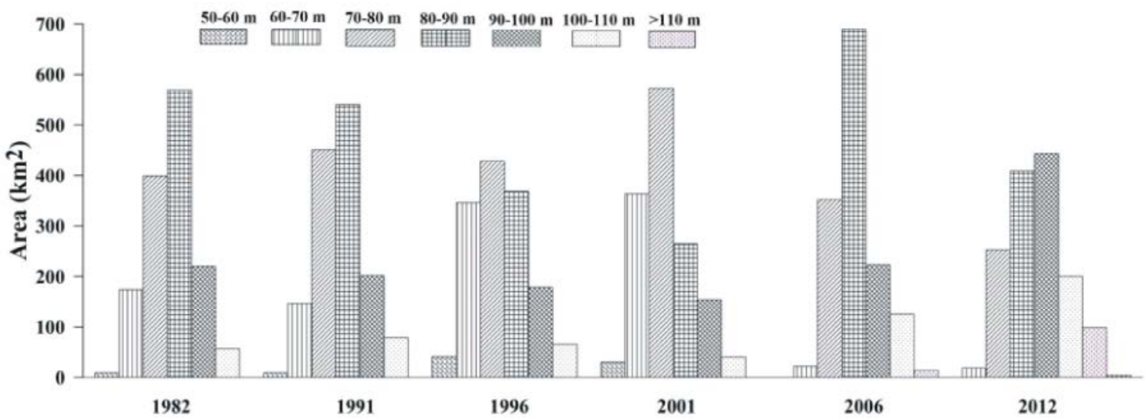

Fig. 6: Distribution of depths to groundwater in the study area for six selected years

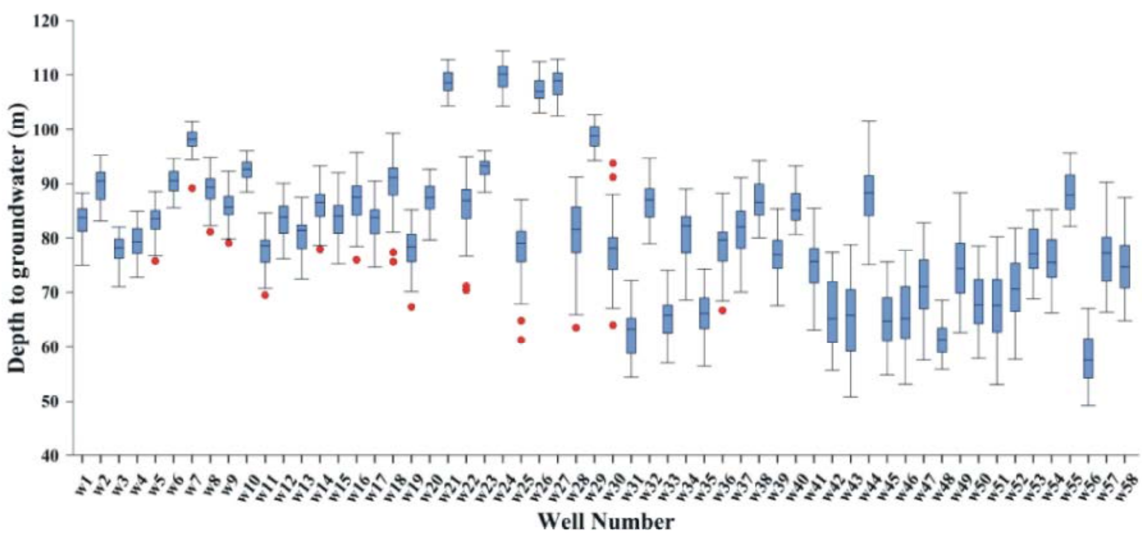

Fig. 7: Box plot of observed depth to groundwater for 58 observation wells

of 0.85 m/yr. As a result, the mean water table shows rising trend with average of 0.22 m/yr. In the main, the depth to groundwater in the region was between 80–90 m in 1982 and 1991, 70–80 m in 1996 and 2001, 80-90 m in 2006 and 90–100 m in 2012 (Fig. 6).

Fig. 7 shows box plot of observed yearly depth to groundwater over the study area for 58 observation wells. It is apparent from Fig. 7 that groundwater level fluctuations of wells W1 to W27 are more stable in comparison with wells W28 to W58. Wells W1 to W27 are located near the Platte River in southern parts of the region (Fig. 1) and probably recharged by this river that lead to maintain the invariable groundwater levels. Meanwhile, the outliers can be seen in the groundwater levels of several wells that located near the Platte River, e.g., W5, W7, W8, W18 and W25. These outliers are caused by rapid rise of the groundwater due to occurrence of groundwater recharge that strengthens the assumption of groundwater recharge by the Platte River. Although temporally the depths to groundwater in southern parts of the region maintain stable with Platte River recharge, but spatially the depth to groundwater in these parts in order to extraction of huge quantities of the groundwater resource for agriculture purposes are relatively deep that make essential the water management strategies in southern parts of the region to achieving appropriate water use. On the other hand, wells W28 to W58 that are located in the northern parts of the study area encounter a considerable groundwater fluctuation. This is due to occurring groundwater recharge during rainy season and depleting groundwater resources during dry season for agriculture purposes.

CONCLUSION

The optimal method for fitting theoretical variogram models namely, spherical, circular, Gaussian and exponential, to the experimental variogram was selected from among fitting methods (including ordinary least squares (OLS) and weighted least squares (WLS)) using yearly depth to groundwater data of 58 observation wells during period of 1982-2012. Histograms and Shapiro – Wilk test indicated presence of normality in the depth to groundwater of all the wells. Cross-validation was used to compare the different theoretical variogram models fitted by fitting methods. Based on the goodness-of-fit criteria (RMSE and correlation coefficient), the circular model fitted by WLS method was selected as the best method for ordinary kriging technique. Measures of uncertainty indicate that circular model fitted by WLS method had the lowest standard deviation of estimation errors (5.17) m and narrowest 95% prediction interval ($Z^* \pm 1.34$). The nugget-to-sill ratio of<0.25 for the circular model fitted by WLS method indicated that the depths to groundwater of the study region have strong spatial dependence. Spatially the depths to groundwater generally increase from the northwestern parts of the region to its southeastern and the deepest groundwater levels are located in a zone in the southeastern parts. Excessive groundwater extraction is the main cause of the decline in the water table, but since most of the croplands in the region are irrigated with center pivots, this procedure resulting in the small fluctuations of depth to groundwater in the past 30 years. The mean water table shows rising trend with average of 0.22 m/yr over the entire of the study area. Furthermore, groundwater fluctuation was found to be comparatively small for the wells located nearby the Platte River in southern parts, due to groundwater recharge occurring by this river that lead to maintenance of the invariable groundwater levels. Overall, although spatially and temporally the depths to groundwater are stable during the past 30 years, but are relatively deep over the whole study area that make essential the water management strategies in the region. As a result, the performance of geostatistical techniques such as kriging in spatial modeling of groundwater levels can be helpful in identifying critical areas that are suffering from deteriorating groundwater level in aquifers, which in turn addresses the necessary need to implement adequate water saving as well as groundwater intensification techniques such as rainwater harvesting and groundwater recharge.

ACKNOWLEDGEMENTS

We are very thankful to USGS for providing necessary groundwater-level data for the present study.

REFERENCES

1. Theodossiou, N. and P. Latinopoulos, 2006. Evaluation and optimisation of groundwater observation networks using the Kriging methodology. Environmental Modelling & Software, 21(7): 991-1000.
2. Zhang, Q.F., Y.M. Wang and Y.F. Xu, 2006. Seepage analysis of landfill foundations in Shanghai Laogang Landfill Phase IV. Journal of Hydrodynamics, Ser. B, 18(5): 613-619.
3. ASCE, 1996. Hydrology Handbook. second ed ed., New York: American Society of Civil Engineers (ASCE).
4. Griffith, D., 1987. Spatial Autocorrelation: A Primer (Association of American Geographers, Washington, DC). C.
5. Tobler, W.R., 1970.A computer movie simulating urban growth in the Detroit region. Economic Geography, pp: 234-240.
6. Krige, D., 1951. A Statistical Approach to Some Mine Valuation and Allied Problems on the Witwatersrand: By DG Krige. University of the Witwatersrand.
7. Olea, R. and J. Davis, 1999.Optimizing the High Plains aquifer water-level observation network. KGS Open File Report, 1999, pp: 15.

8. Prakash, M. and V. Singh, 2000. Network design for groundwater monitoring - a case study. Environmental Geology, 39(6): 628-632.
9. Ahmadi, S.H. and A. Sedghamiz, 2008. Application and evaluation of kriging and cokriging methods on groundwater depth mapping. Environmental Monitoring and Assessment, 138(1-3): 357-368.
10. Kumar, D. and S. Ahmed, 2003. Seasonal behaviour of spatial variability of groundwater level in a granitic aquifer in monsoon climate. Current Science, 84(2): 188-196.
11. Sun, Y., SD. Kang, F. Li and L. Zhang, 2009. Comparison of interpolation methods for depth to groundwater and its temporal and spatial variations in the Minqin oasis of northwest China. Environmental Modelling and Software, 24(10): 1163-1170.
12. Li, L. and P. Revesz, 2004. Interpolation methods for spatio-temporal geographic data. Computers, Environment and Urban Systems, 28(3): 201-227.
13. Machiwal, D., A. Mishra, M.K. Jha, A. Sharma and S.S. Sisodia, 2012. Modeling short-term spatial and temporal variability of groundwater level using geostatistics and GIS. Natural resources research, 21(1): 117-136.
14. Ribeiro Jr, P.J., O.F. Christensen and P.J. Diggle, 2003. geoR and geoRglm: software for model-based geostatistics. in Proceedings of DSC.
15. Irmak, S., I. Kabenge, K.E. Skaggs and D. Mutiibwa, 2012. Trend and magnitude of changes in climate variables and reference evapotranspiration over 116-yr period in the Platte River Basin, central Nebraska–USA. Journal of Hydrology, 420: 228-244.
16. Althouse, L.A., W.B. Ware and J.M. Ferron, 1998. Detecting Departures from Normality: A Monte Carlo Simulation of a New Omnibus Test Based on Moments.
17. Kitanidis, P., 1997. Introduction to geostatistics: applications in hydrogeology. Cambridge University Press.
18. De Smith, M.J., M.F. Goodchild and P.A. Longley, 2006.Geospatial Analysis-a Comprehensive Guide. 3rd edition ed. UK: Troubador Publishing Ltd, Leicester.
19. Cressie, N.A.C., 1993.Statistics for Spatial Data. New York: John Wiley & Sons.
20. Cressie, N., 1985. Fitting variogram models by weighted least squares. Journal of the International Association for Mathematical Geology, 17(5): 563-586.
21. Cleveland, W.S., 1979. Robust locally weighted regression and smoothing scatterplots. Journal of the American statistical association, 74(368): 829-836.
22. Zhang, R., Theory and application of spatial variation. 2005, Beijing, China: Science Press.
23. Isaaks, E. and R.M. Srivastava, 1989. An introduction to applied geostatistics. Press: New York Oxford Univ.
24. Liu, D., Z. Wang, B. Zhang, K. Song, X. Li, J. Li, F. Li and H, 2006 Duan Spatial distribution of soil organic carbon and analysis of related factors in croplands of the black soil region, Northeast China. Agriculture, Ecosystems & Environment, 113(1-4): 73-81.